Vol. XXVIII, No. 5 Whole No. 241

MAY, 1928

SCHOOL SCIENCE AND MATHEMATICS

FOUNDED BY C. E. LINEBARGER

A Journal for all SCIENCE AND MATHEMATICS TEACHERS

CONTENTS:

TURTON & WARNER, *Publishers*
Publication Office: 404 N. WESLEY AVE., MOUNT MORRIS, ILLINOIS
Business Office: 2055 EAST 72ND PLACE, CHICAGO, ILLINOIS
Editorial Office: 7633 CALUMET AVE., CHICAGO, ILLINOIS

Published monthly, October to June, Inclusive, at Mount Morris, Illinois *Price, $2.50 Per Year: 35 Cents Per Copy*
Entered as second-class matter March 1, 1913, at the Post Office at Mount Morris, Illinois, under the Act of March 3, 1879

"We cannot teach anybody anything unless he is interested."
"To make a thing interesting doesn't mean it must be easy."

—*Burdette R. Buckingham*

CONTENTS *for* MAY, 1928

No Numbers Published for JULY, AUGUST AND SEPTEMBER

School Science and Mathematics

A Journal for All Science and Mathematics Teachers

Published Monthly except July, August and September.

PRICE. The subscription price is Two Dollars and Fifty Cents a year; Canada $2.75; foreign countries $3.00; single copies 35 cents.

ALL REMITTANCES should be made payable to the order of **School Science and Mathematics. Remittances should be made by Post Office Money Order, Express Order, or Bank Draft.** If personal checks are sent, please add five cents for collection. Remit to **School Science and Mathematics, 2055 East 72nd Place Chicago.**

CHANGE OF ADDRESS. Subscribers should send notice of any change of address before the 12th of each month; otherwise they are held responsible for magazine, sent to their former address, and no duplicate copies will be sent except on payment of 35 cents for each copy. Send notices to **School Science and Mathematics, 2055 East 72nd Place, Chicago.**

MISSING NUMBERS will be replaced *free* only when claim is made within thirty days after receipt of the number following.

REPRINTS, if desired, must be ordered in advance of publication. Reprints of leading articles will be printed as ordered, the actual cost (with cover, if desired) to be paid for by the author.

MANUSCRIPTS. Contributions on Science and Mathematics Teaching are invited. Articles must be written on one side of the sheet only. All illustrations must be drawn or written in jet black on a separate sheet of manuscript. Contributors are requested to write scientific and proper names with particular care. Manuscripts should be sent to the Editor of **School Science and Mathematics, 7633 Calumet Ave., Chicago,** or to the proper departmental Editor. Books and pamphlets for review should be sent to the Editor.

Entered as Second-Class Matter March 1, 1913, at the Post Office at Mount Morris, Illinois, under the Act of March 3, 1879

SCHOOL SCIENCE AND MATHEMATICS

VOL. XXVIII No. 5 MAY, 1928 WHOLE No. 241

THE MANUFACTURE OF BIOLOGICAL PRODUCTS.*

BY L. T. CLARK,

Research Department of Parke, Davis & Co., Detroit, Mich.

Instruction in bacteriology in preparatory schools is a natural sequence to the greater application of principles of the science to our constantly improved standards of living. Advancement in sanitary and health measures in the nation, state, city, village and rural community is largely based on these principles. Sanitary storage and handling of foodstuffs, the disposal of wastes, the protection of the individual against contagion in the home and in the community, as well as the benefits derived from friendly species, have created an imperative need for a more extensive dissemination of knowledge of bacteria. We are not far removed from the time when many educators considered bacteriology as an advanced branch of botany and were content with the isolation and classification of bacterial species. The more extensive application of the science has brought about a realization that mankind is more concerned with what bacteria do than with their proper name, age, size and former habitat. Such necessity has made biological chemistry a very helpful ally.

The present status of the science in the class room, factory, clinical laboratory, farm and garden, in the home, and at the bedside is made possible largely through the extension of courses in the preparatory schools. Advancement in teaching the science has been stimulated in both school and university by constantly increasing demand for graduates who have completed courses in bacteriology and bacteriological technic.

*Read before the meeting of the Biological Section of the Central Association of Science and Mathematics Teachers, November 25, 1927. The presentation of the paper was followed by two reels of films "How Biological Products are Made."

Laboratories where biological products are made represent only one of the many activities which require the service of individuals trained in the science of bacteriology.

In this brief presentation, consideration of that branch of applied bacteriology, which attaches to the manufacture of biological products alone, will be attempted. It is worthy of note that the present day application of the science is made possible by the vast amount of research in Bacteriology, Chemistry and Medicine which has been accomplished during the past thirty odd years. In this age of scientific progress, research is largely measured by the practical application of its achievements to the needs of mankind.

Since bacteria are minute single celled plants, the soil or culture medium upon which the various species will thrive must receive our first attention. Much depends upon the proper selection of those chemical constituents, which in combination, will promote growth of the organisms in quantity and of such qualities as will maintain uniform standards of activity in the by-products of growth.

The composition of the culture medium best adapted to each species of bacteria has been the object of much research. Comparatively little progress has crowned such effort from the standpoint of chemical analysis. Most species are grown on media containing peptone and meat bouillon as their most essential ingredients, thus making exact chemical analysis of the products of growth very difficult if not quite impossible, because of the numerous chemical elements contained in peptones and bouillon. More particularly is this true of the combinations of Nitrogen which are quite impossible to analyze quantitatively.

It is noteworthy to mention here that a synthetic culture medium has recently been perfected by Drs. Seibert and Long of the University of Chicago. It contains Nitrogen in a single form, —asparagin. This medium contains neither peptone nor bouillon and is subject to definite chemical analysis. This synthetic medium is adapted to the growth of the tubercle bacillus and closely related acid fast species. This achievement has made possible a gigantic research project, which is now being conducted by the Research Committee of the National Tuberculosis Association under the able leadership of Dr. Wm. Chas. White of Washington, D. C. As a result of its use the by-products of the growth of the tubercle bacillus can be separated chemically into definite substances. To what extent this discovery will help simplify

other culture media remains to be determined.

In sequence, the selection of cultures of bacteria for use in production is of greatest importance. This step involves the isolation, identification, maintenance, and evaluation of a large number of strains representative of the various species,—a never ending process. Herein nature's laws governing species and individual variation play a very important role. We differentiate between the oak and the elm by differences in general contour, branch formation, leaf and bark. Microscopical study of bacterial species unfortunately does not afford similar means of recognition and differentiation. In the absence of botanical methods for classification,—the source, chemical changes in culture media, pathogenicity, and serological reactions are applied to the culture to establish its true identity. These technical procedures are merely stepping stones to the real measure of value—that is, the specific activity of each culture used in the development of biological products.

The diphtheria organism, to be suitable, must produce a strong toxin in culture media which in turn will stimulate the horse to formation of a highly potent antitoxin. Strains of typhoid bacilli possessing strong antigenic qualities must be chosen for making the bacterial vaccine used in prophylactic vaccination against typhoid fever. Strains of the same species vary widely in these and other characteristics. This matter of strain selection is considered of such importance as to prompt officials of the United States Public Health Service to issue regulatory measures to manufacturers with respect to the strains which shall be used in certain products. In fact the Hygienic Laboratory distributes cultures, to all manufacturers alike, for the production of some of the serums and vaccines. Too much emphasis cannot be given this very important step in biological manufacture. Many investigators believe that environment exerts a direct influence on the specific activity of strains of a given species. Several of the commoner species are found in more than one pathological condition in a single species, and in more than one species of animal. To what extent the specific antigenic characteristics of a species of bacteria may be modified by such environmental variation, is open to some conjecture, inasmuch as available methods for measuring antigenic power give only relative values.

Likening the above to the gardener's methods of growing the more highly organized plants,—the appropriate soil and carefully selected seed are merely preliminary steps to a successful harvest.

Then follows the development of the mass cultures under atmospheric and temperature conditions accurately controlled to give maximum results. The harvest is measured, not so much by quantity as by quality, which compels us to now turn to standards of evaluation. Only a comparatively small number out of the entire list of biological products are as yet subject to a definite standard of potency or activity. This limited number includes diphtheria, tetanus, scarlet fever and botulinus toxins and antitoxins. Typhoid Vaccine, to a lesser degree of accuracy, and certain of the so-called antibacterial serums are tested for potency by comparison with standard test serums under U. S. Hygienic Laboratory control. The lack of definite standards of potency for other biological products should not be construed as evidence of the absence of effort to devise such standards. In fact, research, directed at the streptococcus group alone, has received the time and effort of a veritable army of bacteriologists over a long period of years. That such effort has not been without merit is evidenced by the discovery of a method for standardizing scarlet fever streptococcus toxin and antitoxin. This feat, superimposed on the earlier work of others in the same field of endeavor, was recently accomplished by the Drs. Dick of Chicago. This discovery of a "measuring stick" for products of one member of the dread streptococcus group opened the door to further successes with that large group of organisms. We are now on the threshold of similar results with the specific streptococci causing erysipelas, child bed fever, rheumatism and measles.

The value of such discoveries was anticipated very early in the search for specific substances of bacterial origin with which to combat infectious disease. At a meeting of the International Congress of Hygiene at Budapest in August, 1894, Roux, Aronson and Behring announced the discovery of diphtheria antitoxin. Their announcement was received amid cheers and a demonstration, the equal of which has never been witnessed at a gathering of scientists. Staid bearded men of mature years threw their silk hats to the ceiling, climbed into their chairs and shouted with joy. They realized that the dread scourges due to the spread of a minute microscopical plant, the diphtheria bacillus, would soon be brought under control. Many of those men who witnessed that demonstration have lived to see diphtheria lose its former epidemic status and in some municipalities practically wiped out through the use of Diphtheria Antitoxin, the Schick test and diphtheria toxin-antitoxin mixture—biological products

amenable to definite standards of potency and activity.

During the present age of scientific accomplishment, which has been styled the "chemical age," the addition of new biological specifics to the physician's armamentarium are being accepted very much as a matter of course. Even so, the discovery of diphtheria antitoxin marked the beginning of an epoch of unprecendented biological research punctuated with noteworthy discoveries. In 1898 Tetanus antitoxin became a reality; soon after, the first antibacterial serum was demonstrated to have some therapeutic value against the streptococcus; 1906 marked the advent of bacterial vaccines and typhoid fever epidemics lost their former dread; in 1913 further aids in the control of diphtheria were introduced in the form of the Schick test and toxin-antitoxin mixture. Three years later, 1916, Pollen Extracts for the diagnosis and control of hay fever were made available to the medical profession; in the early years of the World War a combination antitoxin in a single dose against both gas gangrene and tetanus was found to be of value; and in 1924 the complex streptococcus group lost one of its number to definite standardized products, scarlet fever streptococcus toxin for the Dick test and for prophylaxis, and antitoxin for prevention and cure of scarlet fever.

It may be predicted that the researches now in progress with other members of the streptococcus group, namely on those strains believed to be the cause of infantile paralysis and sleeping sickness by Rosenow, Evans, erysipelas by Ameoss and Birkhaug, child bed fever, by Lash and Kaplan, rheumatism by Small, Birkhaug and measles by Ferry and Fisher will soon result in products of definitely proved value in the control of those diseases.

Progress toward the mastery of Tuberculosis, Pneumonia, Influenza and other respiratory diseases and undulent or intermittent fever continues to be slow, and at best, discouraging. These diseases for which specific biological products of definite value have not been perfected, now occupy prominent places in the field of Medical Reséarch.

In conclusion it may be stated that during the past thirty odd years applied bacteriology has accomplished much in assisting mankind in its never ending struggle against disease. The student in the preparatory school is an important medium through which the interpretation of sanitary measures becomes more certain. In the general scheme of preventive and curative

medicine the commercial biological laboratory is filling an important part by making available under exacting Government control, specific biological products that conform to definite standards of potency, uniformity, purity, and quality.

FROM THE SCRAPBOOK OF A TEACHER OF SCIENCE.

By Duane Roller,
Calif. Institute of Technology, Pasadena.

So, naturalists observe, a flea
Has smaller fleas that on him prey;
And these have smaller still to bite 'em;
And so proceed *ad infinitum.*
—*Jonathan Swift, English satirist, in "Poetry, a Rhapsody."*

I have spent much time in the study of the abstract sciences; but the paucity of persons with whom you can communicate on such subjects disgusted me with them. When I began to study man, I saw that these abstract sciences are not suited to him, and that in diving into them, I wandered farther from my real object than those who knew them not, and I forgave them for not having attended to these things. I expected then, however, that I should find some companions in the study of man, since it was so specifically a duty. I was in error. There are fewer students of man than of geometry.—*Blaise Pascal, French philosopher and mathematician.*

Angling may be said to be so like the mathematics that it can never be fully learnt.—*Izaak Walton in "The Compleat Angler."*

Let not things, because they are common, enjoy for that the less share of our consideration.—*Pliny the Elder, Roman naturalist and author of the first century, A. D., in "Natural History," a work which Curvier characterized as one of the most precious monuments that antiquity has left for us.*

The second law of thermodynamics, of all the generalizations of physics, is certainly the most deeply seated in the common sense of all men, and one of the most humorous of children's verses refers to the man whose wondrous wisdom enabled him to circumvent it by "direct repair":

There was a man in our town
And he was wondrous wise
He jumped into a bramble bush
And scratched out both his eyes.
And when he found his eyes were out
With all his might and main
He jumped into another bush
And scratched them in again.

—*W. S. Franklin and Barrie MacNutt in "Mechanics and Heat," a textbook which every teacher of physics should read.*

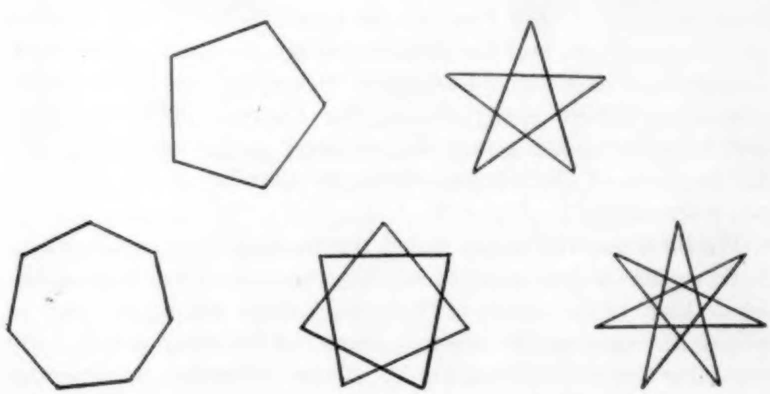
Laws are not masters but servants, and he rules them who obeys them.—*Henry Ward Beecher, American preacher and lecturer.*

THE REGULAR STAR SOLIDS.

By Gertrude V. Pratt,
Mount Clemens, Mich.

In considering the regular star solids, perhaps it would be wise to take up first the regular star polygons, otherwise known as the "multiple-interwoven triangles." The pentegram, or the triple-interwoven triangle, is the only star polygon which we find in the star solids. This polygon was used by the Pythagoreans as a symbol of recognition between the members of the school about the seventh century B. C. and was called by them "Health." The star polygons were known by both Pythagoras and Boethius, and came down to the English speaking people through an able English translator, Adelard of Bath.

If we take n points equally spaced on a circle, and connect them, taking the points in succession in the same direction, we will have a regular polygon. If we connect the points as they come, we have a polygon of the first sort. If we connect every other point, every third, or every fourth, etc., we have polygons of higher order, star polygons. However trial proves that we cannot construct star polygons from any number of equally distant points. There are as many kinds of regular n sided polygons as there are numbers relatively prime to n from 1 to $(n\text{-}1)/2$. Therefore there can be formed only one polygon (of any sort) from three points; one from four points; two from five; one from six; three from seven; and so forth. Polygons of the second sort have their centers doubly, triply, etc., enclosed, according to the number of times the line drawn from the center to any vertex cuts the sides of the polygon, that is



the sides of the triangles that go to make up the polygon. The nucleus is a polygon of the first sort.

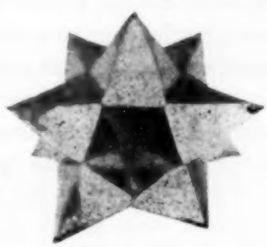
We have five regular polyhedra of the first sort, those constructed in the third semester Geometry, i. e. Solid Geometry, known as the Platonic Solids. These solids, however do not belong to Plato, three being due to the Pythagoreans, (the cube, tetrahedron, and dodecahedron) while the icosahedron and octrahedron are due to Thaetetus. The centers of these solids are singly enclosed by the bounding faces. Just to recall our knowledge of these solids, and to prepare us for the discussion of the star solids, we might refresh our minds concerning the properties of the so called Platonic polyhedra. The cube or hexahedron, possesses six faces, all squares, intersecting in threes in eight solid angles. Each face angle contains ninety degrees. The tetrahedron possesses four triangular faces, which intersect in threes, forming four solid angles, each face angle of which is sixty degrees. The dodecahedron is composed of twelve pentagons, which intersect in threes to form twenty solid angles, each face angle of which is one hundred and eight degrees. Twenty equilateral triangles form the icosahedron, intersecting in fives to form twelve solid angles, each face angle of which contains sixty degrees. The octahedron has eight faces each of which is an equilateral triangle. These triangles intersect in fours to form six solid angles. Other combinations of regular polygons will not make regular solids of the first sort because the sum of the face angles at any vertex either equals or exceeds three hundred and sixty degrees, or a plane surface.

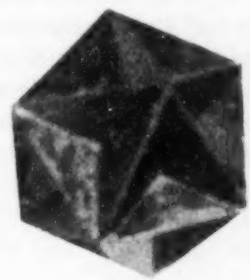
The four regular star polyhedra, in which the center is multiply enclosed are known as the Kepler-Poinsot solids, Kepler having discovered three, and Poinsot the fourth. They have regular polygons as faces, and the solid angles are all equal. They may be inscribed in spheres, and hence in regular polyhedra of the first sort. Their nuclei are regular Platonic polyhedra also, and therefore spheres may be inscribed in the solids, tangent to the faces of the nuclei, which are portions of the faces of the star solids.

We have another group called the Archimedean solids, which have regular polygons as faces, but these faces are not all the same kind of polygons. The solid angles are equal, but of course the face angles are not, since the polygons which form the faces are of different kinds. These are called semi-regular

or discontinuous solids, and are formed from the Platonic polyhedra in much the same way in which the regular star polyhedra are formed.

The first solid with which I shall deal, and which is perhaps the most easily constructed, is the small stellated dodecahedron, called by the Germans the "zwölfeckige Sternzwölfflach." We construct this solid by stellating the faces of the Platonic dodecahedron. That is, we take a certain face as a base, and continue the planes of the faces contiguous to the base until they meet in a point. This forms a pyramid which has for faces equal isosceles triangles, and for a base the pentagon that is the face of the dodecahedron. From the relation of angles, we see that these five isosceles triangles have base angles equal to





seventy two degrees, and vertical angles equal to thirty-six degrees. If we construct such pyramids as we have described on each of the twelve faces of the dodecahedron, we will obtain the required solid. The polyhedron thus formed is bounded by twelve pentegrams, which intersect, five in a group at twelve points, and which intersect in thirty edges. The small stellated dodecahedron may be inscribed in an icosahedron, and may be developed from it, and the center, which we see to be true because of this previous construction, is a dodecahedron. The center of the solid is doubly enclosed by the bounding faces.

Perhaps the next one which we should consider, is the great dodecahedron, known to the Germans as the "sterneckige Zwölfflach," because of its relation to the solid which we have just discussed. It is the reciprocal of the small stellated dodecahedron, and may be constructed by placing inverted pyramids on each corner of the Platonic dodecahedron. These pyramids are determined by the intersection of the same planes we used in the preceding construction, using the inverted pyramid developed on the vertex, instead of the upright one on the face of the dodecahedron. In this case, therefore, the pyramid which we use has an equilateral triangle as a base, and three isosceles triangles as faces, with base angles now equal to thirty six degrees, and vertical angle equal to one hundred eight degrees. Now, if we place the pyramid which we have just found on the small stellated dodecahedron so that its vertex is on the vertex of the nucleus, and its other three corners on the vertices of the small stellated dodecahedron, we see that it fits perfectly provided, of course, that we have used the same sized nucleus, and we see again the reciprocal relation. This is the easiest actual construction, and the relation of the solid thus obtained to the dodecahedron which is its nucleus is very nicely shown. However the theory may be more easily explained thru its relation to icosahedron. Supposing we take a vertex of the icosahedron, and pass diagonals thru the solid. Take two equal diagonals, and pass a plane thru them. This plane will cut the solid in apentagon, of which the aforesaid diagonals are diagonals and of which the sides are edges of the icosahedron. One such plane may be very easily seen from the accompanying photograph, and explains, in a measure, the name of the solid. The inverted pyramids are also visible. Now we can pass twelve such planes thru the icosahedron. These are the faces of the great dodecahedron, intersecting in thirty edges, and in pairs of five, in twelve vertices. The solid may be inscribed in an icosahedron; its nucleus in a dodecahedron, and the center is triply enclosed by the faces.

The solid which follows the great dodecahedron, and may be constructed from it, is the great stellated dodecahedron, known to the Germans as the "zwanzigeckige Sternzwölfflach." It is in theory developed by stellating the faces of the great dodecahedron, that is continuing the faces until they meet in a point. Thus we would have twenty pyramids erected over the inverted pyramids of the great dodecahedron. The simplest construc-

tion, however, seems to be from that obtained by using the Platonic icosahedron as the nucleus of the star solid. Supposing we take a vertex, as in the preceding construction, and draw diagonals of the solid, passing a plane thru these diagonals, which intersects the solid in the aforesaid pentagon. Now, let us continue the edges of this pentagon until they meet in a point. This point is the vertex of our solid, and the triangle thus formed is one of the faces of the pyramid for which we are searching. Each triangle is isosceles, the base angles, being seventy-two degrees, as in the first construction, and the vertical angle, thirty-six degrees. This time, however, our base is triangular, and fits the face of the icosahedron with which we started. If we erect twenty such pyramids on the twenty faces of our nucleus, we will obtain the great stellated dodecahedron. Its twelve faces are pentegrams, which intersect in thirty edges, twenty vertices, three faces to a vertex. It may be inscribed in a dodecahedron, and its nucleus, as seen from this last construction, is an icosahedron. Its center is quadruply enclosed.

The fourth, and last regular star solid, is called the great icosahedron, or "sterneckige Zwanzigflach." It has twenty equilateral triangles as faces, which intersect in fives to make twelve vertices, and which also intersect in thirty edges. It is inscribable in an icosahedron, whose center is septuply enclosed by its faces, and has a dodecahedron as nucleus, as will be seen from the construction. If we take a dodecahedron, and draw the diagonals from one vertex, as we did in the icosahedron, and then pass a plane thru two of the equal diagonals and continue the plane until it intersects the edges of the dodecahedron continued, we find that we will have an equilateral triangle, which is one of the twenty faces of the star solid. We see then that the faces will intersect each other, and will form on each face of the dodecahedron which we took as a nucleus, a queer sort of a pyramid, which has for edges continuations of the edges of our original solid, and as vertices their intersections. But, the base of this pyramid, instead of being the face of the dodecahedron, as in our first construction, is a pentegram, formed on the face of the above mentioned dodecahedron. The great icosahedron is called the reciprocal of the great stellated dodecahedron.

These, then are the regular star solids, and are the only ones which may be constructed. It is impossible to obtain any from either the hexahedron, the octahedron or the tetrahedron.

ATOMS, MOLECULES AND IONS.*

By James B. Conant,

Harvard University, Cambridge, Mass.

An expert has been defined as an ordinary man away from home giving advice. Even under this charitable interpretation, I cannot qualify as an expert on the subject about which I am to speak. On the contrary, I am merely one of the many chemists who have endeavored in the last few years to keep abreast of the times and to discover how many of the new discoveries in physics and chemistry really affect their own bailiwicks.

These have been trying times for those of us who are not specialists in the field of chemical physics. We have heard so many revolutionary statements and so many heated arguments for and against certain electronic theories that at times it seems impossible to put one's feet on solid ground. I propose this afternoon to pass in review from the point of view of an ignorant and skeptical chemist some of the new discoveries and new theories in physics. If there is any originality in my treatment of the material, it will rest solely in the fact that I shall try to emphasize the experimental data and to contrast that which seems to be reasonably certain with what must be taken as only possibly true. This will be particularly the case with regard to the so-called electron theory of valence and the Bohr model of the atom. These excellent scaffoldings which have been so useful to the physicist and to some extent to the chemist, are now being somewhat rebuilt under the influence of Schrödinger.

Atoms.

The new physics and chemistry in many ways center around the periodic table. Instead of assigning to each element a position by virtue of its atomic weight, we now make use of the results of x-ray spectroscopic investigations and arrange the elements according to their atomic numbers.

---

*Presented at a joint meeting of the New England Association of Chemistry Teachers and the Eastern Association of Physics Teachers, at the East Boston High School, Boston, Mass., on May 14, 1927. Reprinted from the Journal of Chemical Education, January, 1928. Acknowledgment is also made to the Journal of Chemical Education for the plates for all illustrations used in this article.

I shall not attempt to go into even the elementary theory back of the x-ray spectrometer, although the accompanying figure (Fig. 1) will show the general principle; indeed, all of you are probably already familiar with it. From the point of view of the elementary student the analogy between the x-ray spectrometer and the spectroscope seems well worth emphasizing. Indeed the x-ray spectra are remarkably simple compared to the visible spectra and their significance much easier to ascertain.

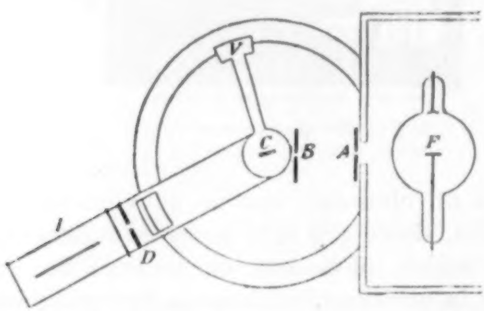


FIG. 1.—X-RAY SPECTROMETER AS USED BY W. H. BRAGG. THE RAYS ARE LIMITED BY TWO LEAD SLITS, *A* AND *B*. THE CRYSTAL IS MOUNTED AT *C*. THE REFLECTED BEAM IS LIMITED BY A THIRD LEAD SLIT, *D*. SOMETIMES A PHOTOGRAPHIC PLATE IS SUBSTITUTED FOR THE IONIZATION CHAMBER, *I*.

It is possible to arrange the elements in such an order that a given pair of lines in the x-ray spectra of each appears to have been progressively and regularly shifted to the left (showing a decreasing wave-length) as one proceeds from the lighter to the heavier elements (Fig. 2). This arrangement of the elements on the basis of their x-ray spectra is found to be exactly like the old series based on atomic weights, *except* that now the pairs of elements, tellurium and iodine, argon and potassium, and cobalt and nickel, are no longer reversed but fall into their proper places.

The chemical properties of the elements are a periodic function of their positions in a series determined by their x-ray spectra. Unlike the old periodic law involving atomic weights, this statement is exact.

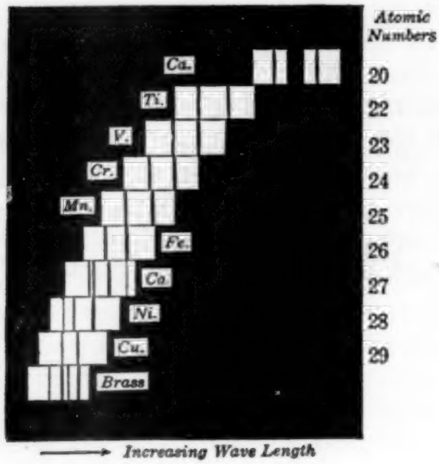


FIG. 2.—X-RAY SPECTRA.

Since not all elements can be conveniently made into anti-cathodes, there are still some experimental gaps in the series which obviously correspond to known elements; and in addition there are a few gaps corresponding to elements which are as yet undiscovered. The number corresponding to the position of the element in this new series, if we take hydrogen as one, is called the *atomic number* of the element. A concise formulation of the new periodic law is, therefore, as follows: *the chemical properties of the elements are a periodic function of their atomic numbers.*

A convenient periodic table based on this new periodic law is shown in Fig. 3. It will be noted that the atomic weight is put under the symbol of the element and the atomic number in the upper left-hand corner of each square. The elements printed in italics and enclosed in heavy lines form the middle of the old long periods. It is evident that the periodicity of chemical properties in terms of the atomic number occurs in such a way that we have (leaving out hydrogen) two short periods of eight elements each, and then two long periods of eighteen elements each. Beyond this point the relationship is less simple, partly because of the complication of the rare earths.

Periodic Table showing Atomic Numbers and Valences

Period	Group 0	Group I	Group II	Group III	Group IV	Group V	Group VI	Group VII	Group VIII
0		1 H (1) 1.008							
I	2 He (0) 4.00	3 Li (1) 6.940	4 Be (2) 9.02	5 B (3) 10.82	6 C (4) 12.000	7 N (5) 14.008	8 O (6) 16.000	9 F (7) 19.00	
II	10 Ne (0) 20.2	11 Na (1) 22.997	12 Mg (2) 24.32	13 Al (3) 26.97	14 Si (4) 28.06	15 P (5) 31.027	16 S (6) 32.064	17 Cl (7) 35.457	
III	18 A (0) 39.91	19 K (1) 39.096	20 Ca (2) 40.07	21 Sc 45.10	22 Ti 48.1	23 V 50.96	24 Cr 52.01	25 Mn 54.93	26 Fe 27 Co 28 Ni 55.84 58.94 58.69
		29 Cu 63.57	30 Zn 65.38	31 Ga 69.72	32 Ge 72.60	33 As (5) 74.96	34 Se (6) 79.2	35 Br (7) 79.916	
IV	36 Kr (0) 82.9	37 Rb (1) 85.44	38 Sr (2) 87.63	39 Y 88.9	40 Zr 91.	41 Cb 93.1	42 Mo 96.0	43 Ma ?	44 Ru 45 Rh 46 Pd 101.7 102.91 106.7
		47 Ag 107.880	48 Cd 112.41	49 In 114.8	50 Sn 118.70	51 Sb (5) 121.77	52 Te (6) 127.5	53 I (7) 126.932	
V	54 Xe (0) 130.2	55 Cs (1) 132.81	56 Ba (2) 137.37	57-71 Rare Earths	72 Hf 180.8	73 Ta 181.5	74 W 184.0	75 Re ?	76 Os 77 Ir 78 Pt 190.8 193.1 195.23
		79 Au 197.2	80 Hg 200.61	81 Tl* 204.39	82 Pb* 207.20	83 Bi* 209.00	84 Po* 210.0	85—	
VI	86 Rn (0) 222.	87—	88 Ra* (2) 225.95	89 Ac* 227	90 Th* 232.15	91 Pa* 230	92 U* 238.17		

FIG. 3.—THE *atomic number* APPEARS BEFORE THE SYMBOL OF EACH ELEMENT AND THE *atomic weight* BELOW. THE NUMBER OF *valence electrons* IS IN PARENTHESES. THE ATOMIC WEIGHTS OF PO, AC, AND PA ARE CALCULATED VALUES. *AT THE PLACES MARKED * ARE TO BE ASSIGNED A NUMBER OF OTHER ELEMENTS, FOR THE MOST PART RADIOACTIVE, WHICH HAVE THE SAME CHEMICAL PROPERTIES AS THE ELEMENT IN THE TABLE BUT SOMEWHAT DIFFERENT ATOMIC WEIGHTS.

THE POSITIVE NUCLEUS.

Now for the interpretation of the experimental data. Rutherford first postulated the existence of a positive nucleus in each atom to explain the scattering of α particles (from radium) by matter. I think we need not depart too far from the attitude of the "skeptical chymist" to consider the reality of this positive nucleus as well established—though I think it would be going much too far to call it a "fact." At least we may say that it seems impossible to correlate a great many different experimental facts without assuming a nucleus in each atom having essentially the mass of the atom and a positive charge.

Rutherford's experiments had led him to believe that there was at least an approximate correlation between the scattering power of an element and its atomic weight. Since he had postulated that the magnitude of the charge on the positive nucleus determined the amount of scattering, he summed up the situation as follows: "The general data available indicate that the value of this central charge for different atoms is approximately proportional to their atomic weights, at any rate for atoms heavier than aluminum."

Moseley undertook his famous work in order to investigate this point by an entirely different and more rigorous method. The result was Moseley's law connecting the wave-length of the x-ray spectral lines and the position of the element in a series (the atomic number). We have just discussed the significance of this discovery in relation to the periodic law. From the point of view of the physicist, the atomic number is the net charge on the positive nucleus, since the present physical explanation of the emission of x-rays leads to an equation involving the magnitude of this nuclear charge.

If the charge on the positive nucleus determines the atomic number of the element (its place in the periodic table), it is conceivable that one could turn one sort of atom into another by modifying the nucleus. The artificial production of such a change is a very doubtful point, as I will mention later, but nature has provided a remarkable series of elements where this process goes on for us. The facts elucidated by a study of radioactivity are among the most important which have led to the new ideas concerning the structure of atoms.

Evidence From the Disintegration of Radium.

The accompanying figure (Fig. 4) shows the position in the periodic table of the disintegration products of radium. Radium having the atomic number of 88 (with the chemical properties of group 2) loses a doubly charged helium atom (alpha particle) and is converted into radium emanation, now called radon (atomic number 86). This member of the rare gas family loses another doubly charged helium atom and becomes radium A of atomic number 84, which in turn becomes radium B with the number 82. These are all so-called *alpha* changes and, as would be expected from the concept of a positively charged nucleus, each loss of two positive charges from this nucleus yields an element of atomic number two lower. On the other hand, a so-called beta change involves the liberation of an electron *from the nucleus.* In terms of our theory the positive nucleus thereby gains a positive charge, and the newly formed element has an atomic number one greater.

LAST TWO ROWS OF THE PERIODIC TABLE SHOWING POSITION OF THE DISINTEGRATION PRODUCTS OF RADIUM

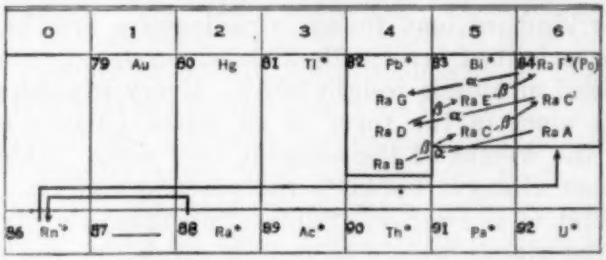


FIG. 4.—THE POSITIONS MARKED BY A STAR (*) ARE OCCUPIED NOT ONLY BY THE ELEMENTS GIVEN BUT BY THE DISINTEGRATION PRODUCTS OF THE THORIUM-URANIUM SERIES. NOTE THAT IN THE CHANGE FROM RN (RADON) TO RA A, THE CHANGE FROM THE ZERO GROUP OF ONE ROW TO THE SIXTH GROUP OF THE *higher* ROW IS EQUIVALENT TO MOVING TWO POSITIONS TO THE LEFT IN THE SAME ROW (α CHANGE). RADIUM F IS ALSO CALLED POLONIUM (Po).

It must, of course, be remembered that all these changes are occurring spontaneously at a rate entirely independent of anything the chemist can do. Some of the changes are so rapid that the life of one of the elements is half over in a few minutes. In many of the cases, the amounts of the substances involved have been too small to see or even weigh. The results are nevertheless beyond dispute, thanks to the accuracy of the physical methods of measuring the characteristics of these elements.

There seems no other way of explaining the remarkable regularities of these radioactive changes except on the basis that we are here dealing with spontaneously exploding nuclei and that the alpha particle and electrons both come from within this tiny nucleus.

ISOTOPES.

The study of radioactivity first revealed to the chemist that a chemical element might be a mixture of two or more substances of the same atomic number but with different atomic weight. A glance at Fig. 4 shows that ordinary lead, radium B, D, and G, all have the same atomic number (82) and, therefore, identical chemical characteristics but *different* atomic weights. These elements are *isotopic* with one another. Isotopes are identical in chemical properties and differ only in atomic

weight, density, and other properties involving atomic mass. It has been shown that samples of certain kinds of lead found in radioactive ores have an atomic weight of practically 206, as compared with ordinary lead of atomic weight 207.2. Every expulsion of a helium atom in the form of an alpha particle should lower the weight of the element by 4 units. There are five such changes between radium and radium G, the end-point of the series. Subtracting these twenty units from the atomic weight of radium gives 206, the atomic weight of radium G. The experimental demonstration of the existence of two kinds of lead, identical in chemical properties and x-ray spectra but different in properties involving atomic mass, was the first proof of the existence of isotopes.

Subsequently, the physicist Aston showed that isotopes are not confined to the last place in the periodic table, but that many of our common elements are mixtures and that their atomic weights are averages. These results were gained through a study of the behavior of individual charged atoms moving rapidly in a highly evacuated tube under the influence of an electric discharge. By determining the effect of electric and magnetic fields on the path of these atoms (as recorded by their effect on a photographic plate) Aston was able to calculate the mass of the atom. This work is remarkable not only because of the significance of the results obtained, but because we have here an experimental study of practically individual atoms; although, to be sure, the action on the photographic plate is due to the action of a multitude of individual atoms following each other on the same path.

Mixtures of isotopes are inseparable by chemical methods, but may be separated with great difficulty by diffusion processes. This has actually been done in the case of mercury and chlorine, but the separation is far from complete, since the isotopes differ but slightly in mass. For all chemical purposes a mixture of isotopes is a constant, definite element.

It is significant that the atomic weights of the individual isotopes are very nearly whole numbers, though the conglomerate of isotopes which we know as a chemical

element may have an atomic weight far removed from an integer. For example, chlorine, whose atomic weight is 35.46, has been shown by Aston to be a mixture of two isotopes of atomic weights 35 and 37 in such proportions that the mean value is 35.46.

Are all the elements composite substances made of some simpler parts, such as hydrogen nuclei and electrons? The fact that the atomic weights were not whole numbers killed a similar attractive hypothesis in Prout's time; can it come to life again now that we know that the atomic weights of the individual isotopes are very nearly whole numbers? Many scientists today answer these questions in the affirmative. Certainly in the case of the radioactive elements we are dealing with composite atoms. If the hydrogen nucleus (often called a *proton)* and electrons are the building stones of the nuclei of all the atoms, the structure of all matter has been reduced to very simple terms indeed. According to this modern Prout's hypothesis, the helium nucleus with net charge 2+ is composed of four protons (4+) and two electrons (2−); other nuclei may be imagined as similarly constructed. There are certain relatively slight discrepancies in mass since the atomic weight of hydrogen is 1.008 and not 1.000, but these can be accounted for in terms of modern physical theory.

The idea that the nuclei of all atoms are composite at once suggests the possibility of breaking down complex atoms into simpler ones—a transmutation of elements. Such a process is going on all the time in the case of the radioactive elements as we have seen, but we know of no method to hasten or retard it. The experiments of Rutherford in which he obtained hydrogen by the bombardment of nitrogen by alpha particles represents in a sense the first artificial "transmutation" of elements. But it is well to remember that the amounts of matter so transmuted were so small as to escape discovery by any chemical method. Furthermore, since a radioactive element was used as the source of energy, the use of the term *artificial* disintegration has been questioned. The sum total of the process is simply this, that the disintegration of a radio-active element under certain condi-

tions can bring about the disintegration of a small amount of another element. Whether or not the change of one element into another can be brought about by other means than by subnuclear forces (for instance, by means of electric currents) is still a much disputed point. No generally accepted case of clearly artificial transmutation is known, though there seems to be no reason to believe that such processes are intrinsically impossible.

Molecules and Ions.

So far, I have dealt chiefly with the structure of the atom and even flirted with the structure of the nucleus. I shall now briefly consider some problems rather more chemical in nature, namely, those concerned with the structure and formation of compounds. By means of the x-ray it has been possible to decide the positions of the structural units in a crystalline solid. In the case of sodium chloride, the results are remarkably simple as shown in the accompanying figure (Fig. 5). A number

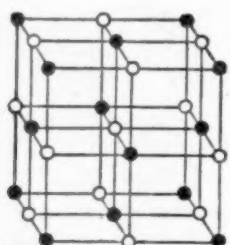


Fig. 5.—Atoms in a crystal of common salt.

of other common salts similarly show a "space lattice" in which there seems to be no grouping of one positive element with another negative element, but rather a uniform distribution of both elements. On the other hand, some inorganic substances such as certain oxides (*e. g.*, Fe_2O_3, As_2O_3, Cr_2O_3, Al_2O_3 and SiO_2) and tin tetraiodide, and probably all organic substances show a quite different sort of structure. These are illustrated in Figs. 6, 7, and 8. It will be noted that here we are dealing with definite groups of atoms, these groups in turn forming a crystalline unit. These groups are in most in-

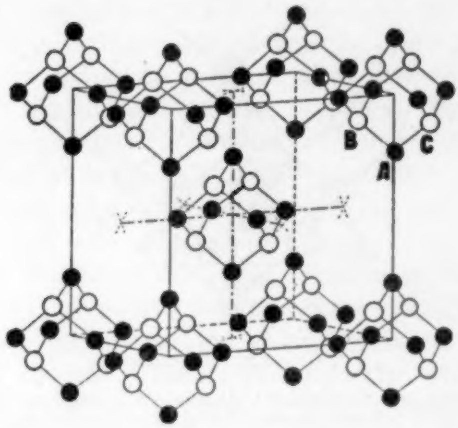


FIG. 6.—ARRANGEMENT OF THE CARBON ATOMS (BLACK CIRCLES) AND NITROGEN ATOMS (WHITE CIRCLES) IN $C_6H_{12}N_4$ (HEXAMETHYLENE TETRAMINE) CRYSTALS.

stances the molecules, and the x-ray has given us a method of demonstrating their existence in a solid with considerable certainty. It is perhaps not out of place to note that some popular writers have mistaken the results with sodium chloride as being the whole story and have stated that modern physics has shown the non-existence of the molecule in solids. Nothing could be farther from the truth.

Returning to the structure of sodium chloride, we can either imagine that the uniformly spaced atoms are sodium atoms and chlorine atoms, or that they are sodium ions or chlorine ions. There are a number of facts which I believe the physicists can bring forward to support the latter contention and in view of what we know of the properties of salts it seems the rational one. The crystal of a salt, like sodium chloride, is thus probably made up of ions held together by electrostatic attraction. When the salt dissolves in water, the ions are loosened and are free to move. The same is true if we melt a salt. Such compounds (often called polar) are to be contrasted with organic substances (and some inorganic substances which are not salts) where we do not have an electrolyte formed on dissolving the material. As far as the evidence at present is available, we

Fig. 7.—Single molecule of $C_6H_{12}N_4$ (hexamethylene tetramine.) The Small white circles represent hydrogen atoms. Perspective.

can say that these non-polar compounds almost always show, on x-ray analysis, a grouping of atoms corresponding to a real molecule.

Valence in Non-Polar and Polar Compounds.

Concerning the forces which hold the atoms together in the molecules of non-polar substances, we are completely in the dark. I say this with all due respect to the brilliant theory of Lewis and to its extension by Langmuir and others. There are some interesting hypotheses concerning the nature of a non-polar union, but they are certainly at present in an entirely different category of probability from the simple points which I am trying to emphasize this afternoon. In regard to the problem of valence in such non-polar molecules as the hydrogen

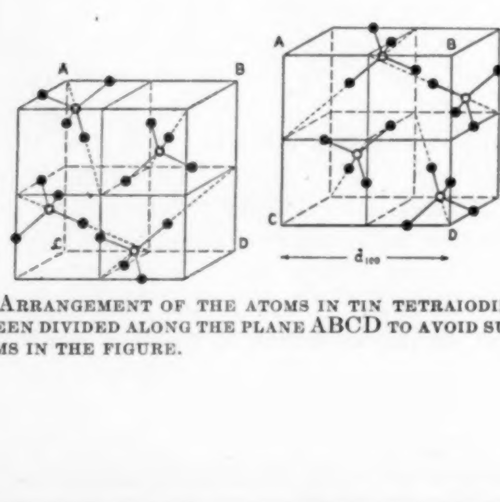


Fig. 8.—Arrangement of the atoms in tin tetraiodide; the unit cube has been divided along the plane ABCD to avoid superposition of the atoms in the figure.

molecule, the carbon dioxide molecule and the methane molecule (and practically all of organic chemistry), the new ideas have given us no sure clue and there is no agreement among the contending theories. To say that the electron theory has "explained valence" is to forget the existence of at least half of all chemistry.

Since in the simple salts we are dealing only with an aggregate of ions, the problem of valence is quite different. Here, if we know the rules relating to the formation of ions from atoms and can find some probable reason for these rules, we can claim to have "explained" this class of compounds. This can be fairly well accomplished in the case of the first twenty elements in the periodic table. Returning to our picture of the atom, it will be evident that in order to have an electrically neutral atom there must be some negative electricity to counterbalance the positive charge on the nucleus. To explain the periodicity in chemical properties and a great many physical and chemical facts, it has been suggested that the negative electricity is in the form of electrons which range themselves in layers or shells; the first shell contains two electrons and the next two shells eight each. The most elaborate and complete theory of this sort is the one put forward nearly fifteen years ago by Bohr and slightly modified by him from time to time. I am sure all of you are familiar with the fundamentals of this theory, and I need but to call them to your minds as regards their chemical significance. The lack of reactivity of helium, neon, and argon is attributed to the fact that in each of these rare gases we have completed groups of shells of electrons, the first having a group of two, the second a group of two surrounded by a group of eight and the third (argon) two large groups of eight surrounding the inner group of two. It was further supposed that this rare gas arrangement of electrons was the most stable and that all other atoms would tend to take on this arrangement by virtue of a gain or loss of electrons. Thus, the lithium atom by losing one electron from its total of three has a structure resembling the helium atom, but carrying a positive charge. This is the lithium ion. Similarly, the sodium ion is like neon, the potassium

ion like argon, as far as the arrangement of the electron is concerned. In the case of the strongly electronegative elements, such as the halogens, it is supposed that they can acquire a sufficient number of electrons to make up a group of eight. Thus, fluorine with seven takes up one electron and becomes the fluorine ion, similar to neon in electronic structure. The sulfur atom, with six electrons, acquires two and becomes the sulfide ion, analogous to argon. The following figure (Fig. 9) shows a diagrammatic scheme indicating the arrangement of

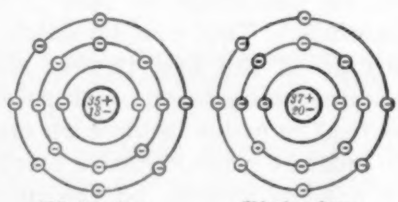


FIG. 9.—ISOTOPES OF CHLORINE.

electrons in the isotopes of chlorine on the basis of this theory. If we were to add one more electron to the outer ring, we should have the chloride ion. While the theory works excellently for polar compounds (that is, salts) of the first, twenty elements, there are considerable difficulties met within its application to the long periods. I shall not deal with these. It is an interesting and useful rule to remember that the elements in the middle of these long periods are the ones that show variable valence and form colored ions.

REVISION OF THE IONIZATION THEORY.

The ionization theory has been revised in the last few years partly as an outcome of the new point of view in regard to the structure of the crystals of simple salts. In the case of such substances as sodium chloride, the undissociated molecule has been discarded. This has been accepted with enthusiasm by almost all. The simple theory of ions made its quantitative calculations on the assumption that the gas laws could be applied to a solution of these charged particles. The new theory of Debye started from the point of view that a solution of a salt

is very different from one of sugar because we are dealing with highly charged particles and we must take into account the electrostatic forces. He has been able to develop a theory which correlates many of the old and apparently hopeless data concerning the so-called degree of dissociation of strong electrolytes. According to the new point of view the change in conductivity of sodium chloride solution on dilution is not due to an increase in dissociation, but to the different electrical conditions prevailing in concentrated and dilute solutions of highly charged particles. Qualitatively, the picture given by the new view of salt crystals and total ionization is really simpler than the old concept. In interpreting the common ion effect with strong electrolytes and the formation of precipitates we should do well nowadays to use an explanation involving the direct combination of ions to form a solid, rather than to introduce another equilibrium involving the now discarded non-ionized molecule. Thus:

Old View.

$$\underset{\text{}}{Ag^+ + Cl^-} \rightleftharpoons \underset{\text{(undissociated molecule)}}{AgCl} \leftrightarrows \underset{\text{(solid)}}{AgCl}$$

New View.

$$Ag^+ + Cl^- \rightleftharpoons \underset{\text{(solid)}}{AgCl}$$

There seems to be no reason, however, to change our ideas concerning the dissociation of weak electrolytes, which have been found to follow quantitatively the predictions made on the basis of an equilibrium involving an undissociated molecule (Ostwald's dilution law). Thus

$$\underset{\text{(undissociated acetic acid)}}{H[C_2H_3O_2]} \rightleftharpoons H^+ + C_2H_3O_2^-$$

Changes in the Physicists' View of the Atom.

In the last year, the status of the Bohr theory of atomic structure has rapidly changed. All admit that it has been of extreme value and probably still is the most satisfactory working hypothesis in many fields. That it represents the real state of affairs, however, seems unlikely

in the opinion of those most competent to know. I am here on even more uncertain ground than in the rest of my talk and will content myself with quoting from a recent paper by one of the brilliant young chemists who is following the most recent advances in physics and is foremost in interpreting the Schrödinger ideas.

This model of the hydrogen atom accordingly consists of a nucleus embedded in a ball of negative electricity—the electron distributed through space. The atom is spherically symmetrical. The electron density is greatest at the nucleus, and decreases exponentially as r, the distance from the nucleus, increases. It remains finite, however, for all finite values of r, so that the atom extends to infinity; the greater part of the atom, however, is near the nucleus—within 1 or 2 Å. . . . The chloride ion may be described in the following words: the nucleus is embedded in a small ball of electricity, consisting of the two K electrons, with a trace of the L and M electrons; surrounding this are two concentric shells, containing essentially the eight L and the eight M electrons.

This is certainly a complex picture and perhaps at present unintelligible to the non-mathematically minded. It should be pointed out that as far as most of the interpretations are concerned which I have given this afternoon, we do not have to modify them essentially even in terms of the very recent ideas. If we define valence electrons not in terms of the Bohr model, but merely as the number of negative charges which the atom can gain or lose in making up the mystic number of eight, we are in a safe position, always with the proviso *that we stick to polar compounds of the first two rows of the periodic table.* For the present this seems to me sufficient for those of us who are not actively engaged in trying to unravel the complexity of atomic structure. The passing of the Bohr atom may well remind us, however, that we should be cautious about being dogmatic in presenting even a very satisfactory theory.

GREENLAND TEACHER WILL INSPECT ALASKAN SCHOOLS.

Vorstander Bugge, principal of the seminarium at Godthaab, Greenland, will study at first-hand the schools for natives of Alaska administered by the United States Bureau of Education. The itinerary of Mr. Bugge was planned by Dr. J. E. Church, Jr., a member of the recent Hobbs Greenland expedition, in cooperation with Bureau of Education officers. It will take him as far north as Nenana, down the Yukon River to Golovin Sound, thence to Nome, and if time permits to Kotzebue and Barrow, returning by way of Little Diomede, Nome, and St. Lawrence Island to Seattle.—*School Life.*

THE TEACHING OF PHYSICS.

BY A. A. BLESS,
Cornell University, Ithaca, N. Y.

On meeting a new class at the beginning of the school year it is my habit to question the students on their ideas of Physics, the scope of the subject and its importance. So far the answers to these questions have been anything but satisfactory. The answers range from the indefinite, all embracing "Well" down to complete silence. It makes little difference whether the student has had Physics before coming to my classes or not; the character of the replies remains unchanged.

The questions I am asking are not for the purpose of confusing the students, although they are very effective in this respect. I believe that a clear idea of the significance and scope of the subject studied, the part it plays in the progress of human civilization, the way in which the growth of the subject is taking place and some of the very elementary laws and theories of Physics should be given to the student during his first year of study of this subject. If my questions are fair, the answers I receive show that the first contact of the student with this science is not very fruitful.

It is not only as a test of previously acquired knowledge that I am interested in the question of the content of Physics. A clear idea of the scope of the subject provides a basis for all later discussions, it is a foundation on which the whole structure of the subject should be built.

It is generally agreed that the textbook plays a very important part in the instruction of high school students and college freshmen. Lecturing is the least satisfactory method of instruction for these students, unless the lectures follow a book very closely. Any attempt on the part of the instructor to depart from the book is doomed to failure, partly because the student has more faith in the "printed word" and partly because of his inability to take adequate notes. The textbook plays therefore a very important part in shaping the ideas of the students.

In order to find out how the modern texts are treating the subject matter of Physics and their idea of its scope I

looked over fairly carefully some forty elementary high school and college texts on this science. The results of this survey are extremely disconcerting and partly, at least, explain the haziness of the notions of the student on the subject.

About fifty per cent of the writers of our texts have no notion of any unity underlying Physics, or if they have, they do not divulge it. Some of them make a feeble and ineffectual attempt to limit the domain of the subject they are about to discuss by prefacing the book with some such hazy and indefinite statements as "Physics deals with some phenomena" or "Physics tells the why and how of certain phenomena." One writer declares that "Physics does not deal with all phenomena occurring in nature but only with few." Very illuminating, indeed! In most cases, however, the writer plunges into the very exciting and interesting definition of the meter or yard without any introductory statements whatever.

From these definitions or lack of them it would seem that Physics deals with an assemblage of phenomena having nothing in common. Mechanics, light, heat, sound and electricity are discussed probably because no other science cares to deal with these topics, the inclusion of these topics in a single course being largely a matter of accident. One text book actually states that Physics deals with matter and such general phenomena as heat, light, electricity, and sound. It is difficult to show more plainly that the writer knew of no reason why these "general phenomena" belong to Physics.

To give due credit to the writers of these texts their treatment is consistent with their notions, or rather, lack of notion of what Physics is. There is no attempt to show the connection between the various divisions of the subject. In fact, each topic is dealt with as if it were a separate entity, having no relation to the rest of the book.

I realize that it is possible to give a very interesting account of a certain subject without defining it. However, an interesting exposition is much more effective if it is logical and coherent. It is easier to understand the subject if one is not led to believe that every paragraph is a separate and complete unit to be memorized as if it had nothing

to do with the rest of the subject. It is much easier for the student to get a better grasp of the elements of a science if he is able to see the unity underlying all phenomena discussed in the book.

The other twenty texts examined are more ambitious. There is an attempt to define the subject and thereby limit its scope and give the book a semblance of unity. Aside from the question of the correctness of the definition the treatment of the subject in these texts differs very little from that of authors who make no attempt to explain what Physics is. The exposition is in no case subordinated or even related to the introductory statements. No reasons are advanced for the inclusion of this or that division of this subject in the light of the definition given, and there is no attempt to show the relation between these divisions. The exposition is, with very few exceptions, as cut and dried as catalogue descriptions. The difference in the textbooks is for the most part so small that I often wonder why the later texts were added to the stock already in existence. Changing a paragraph here and there or introducing a few new ones is hardly a sufficient reason for writing a book.

The definitions of Physics found in these texts are no more satisfying than the rest of the book. Some of the statements are plainly ridiculous. Thus, two writers declare that "Physics is the study of how to measure things." I am sorry for these people: Physics must be very dull for them. Another author states that "Physics is the study of sense perception." Mechanics, sound, heat and light are treated in Physics because they are perceived by our senses. The writer regrets that nature has not endowed us with a sense for perception of electricity. Perhaps getting a hold of a high tension wire would enable one to discover a sense responding to this form of energy. It is a pity that smell and taste have been slighted, for the study of the properties of matter affecting these senses has been neglected by physicists.

The other books are defining Physics as the study of matter, or matter in motion, or the study of matter and energy. To me this view is quite incomprehensible. I doubt whether in Physics we are interested in properties of matter "per se." We do not care for the degree of polish capable of

attainment by various materials, we make no special study of the brittleness of materials, their smell and taste. We study only those properties of matter by virtue of which a resistance is offered to the action of outside forces. These properties are interesting only in so far as they help us to understand the action of energy on matter, and enable us to utilize to greater advantage the forces at our disposal.

Physics may be defined as the study of energy only. It seeks to find the nature and the properties of energy for the purpose of utilizing these properties for the benefit of mankind. The economical aspect of this utilization is the problem of engineering. As it is impossible to make industrial application of energy without the agency of matter, the study of those properties of matter which affect the application and utilization of energy are of great interest to physicists.

This way of looking at Physics distinguishes this science from Biology and Chemistry very sharply. It is true that most natural processes are very complicated and involve changes of matter as well as energy changes. But each component of the process could be separated and analyzed by the means of the science to which this component properly belongs. Thus the growth of a tree is a result of biological, chemical as well as physical changes. Each aspect, however, is clearly distinguishable. The amount of energy, for instance, involved in the growth is essentially a physical problem and could be determined by the methods of this science.

It is true that to confine Physics to the study of energy relations would exclude from our textbooks and from our instruction a number of topics which are still being inserted in an elementary text. Pages on musical scales and transposition of scales could be found in most of our high school and college textbooks. The relation of these topics to energy is so remote that there is no point in teaching them. The cutting out of topics from our books and instruction should really go farther than that. It is an open secret that at least a third of the material usually included in a book is forgotten by the student (if he ever knew it) a few hours after the final examination. It would be a much better procedure to confine our attention to a few

essential principles and have them covered as thoroughly as possible. It never pays to sacrifice thoroughness for volume. Almost every college text deems it its duty to discuss the Carnot cycle and other topics of this caliber. While the importance of these topics introduced at the proper time cannot be denied, they can hardly be taken up with sufficient completeness in an elementary course. It is best to omit them.

The general method of presentation of the material which should be studied is a matter of great importance. While the progress of Physics is going on more or less inductively, the facts in most cases being known before the laws governing these facts are discovered, it would be absurd to present the subject in the same way, to compel the student to memorize a number of disconnected facts, and later to tell him that all these facts are direct consequences of a few general principles. This procedure, however, is followed by most of our books, except that some of them fail to mention at all how these general principles explain the phenomena. To be more specific, all the material treated in the chapters on electricity could be explained by means of the assumed properties of the electrons, just as all the phenomena of heat and the properties of gases are consequences of the law of molecular motion. Instead of forming the basis for the treatment of these chapters and thereby making the presentation of these portions of physics more logical and easily understood the Kinetic Theory of Gases and the modern concepts of electrons are placed in odd parts of the book as just so much more stuff to be memorized by the student. The deductive method of teaching is more effective inasmuch as the number of facts to remember is not as great. This method develops the power of the student to reason out things and extend the general principles to particular cases, a faculty essential in a physicist or engineer. The deductive method should be used whenever possible. The modern advances of Physics are not sufficiently made use of in the treatment of the subject. An effort must be made to show that most, if not all of the material could be explained on the basis of discoveries made recently.

The use of the deductive method in presenting the sub-

ject, contrary to its historical development, does not mean that all historical references are to be omitted. There is nothing so valuable in stimulating the interest of the reader as frequent references connecting the material studied with the lives of men most responsible for the progress in the given field. In fact, a great deal more space and time could be very profitably devoted to topics intended to stimulate interest in Physics and give the student a better appreciation of its importance. The part played by Physics in the progress of human civilization, the dependence of our material progress on the discovery and use of physical laws is, I believe, legitimate material for textbook and class room discussion. It is also quite important for stimulating interest in Physics to show how thoroughly alive this science is. It is a mistake to lead the student to believe that everything in Physics is known, which would be equivalent to saying that this science cannot grow any more. Some of the problems confronting the physicist, the doubts concerning some theories and the inconsistencies in others, the intense work that is going on in that field, the exciting discoveries made recently, all this should be a part of our teaching.

Confining our efforts to the study of the essentials of Physics, deducing these essentials from as few general principles as possible, showing the unity underlying all the laws and divisions of Physics, and introducing material tending to show the importance of the subject, and its fascination would, I believe, enhance the value of our teaching a great deal.

THE SIGNIFICANCE OF TESTS AND TESTING.

Tests give us very valuable information about the child. While accepting the information which they have to offer, the teacher should not neglect other sources of knowledge and use *only* scores of standardized tests. Often this would involve a very serious injustice to the individual child. For instance, never grade in subject matter by the intelligence-test score. It would be almost as fair in certain subjects to grade children by their height and weight as by their score on the intelligence test. Neither should the achievement tests in specific subject matter be the *sole* basis for grading; everything else that the child has done during the year should be equally well considered, to avoid unfairness to him. *Test scores are merely valuable evidence; they are not final judgment.*—From "An Introduction to Educational Measurements" by Norman Fenton and Dean A. Worcester.

SYNTHETIC PROJECTIVE GEOMETRY IN SECONDARY SCHOOLS.

By H. R. Phalen,

St. Stephen's College, Annandale-on-Hudson, N. Y.

At various times in all countries where Euclid is taught there have been efforts to point out its difficulties and to recommend improvements. The question occupied the attention of Clairaut and also of D'Alembert who seemed not to share the general belief that Euclid was a sort of mathematical decalogue which could be neither altered nor amended. D'Alembert especially suggested that text books be rewritten and that the revision be executed by the best mathematical talent of the world and that it be done along the lines of the growth of the subject. The idea met with approval but as is frequently the case the great mathematicians were too busy with research to direct their attention to elementary geometry. The natural result was that the books which reached the hands of the students were for the most part the products of woefully incompetent authors. One brilliant exception was the work by Legendre which, contrary to the fate of most books by writers of ability, enjoyed unique success not only in Europe but also in this country where in the form of Davies' translation it was the standard for several decades.

De Morgan in 1833 decried the methods and material then in use and in his characteristic manner made suggestions which, if heeded, would have put education in the British Isles ahead at least a generation.

In addressing an assembly of teachers in Germany in 1902 Theodore Reye spoke as follows:

"The demand for a fertilization and revival of school teaching through the introduction of new methods is making itself more universally and more stringently felt. On that account a comparison of the old and young branches of geometry does not appear untimely."[1]

The above remarks are in effect a suggestion that synthetic projective geometry might contain inherent elements which would make it worthy of consideration as a part of the secondary school course in geometry. To bring a subject from the graduate department of the university down to the high school appears at first to be a considerable feat of pedagogical legerdemain.

[1]Theodore Reye, *Jahresbericht der Mathematiker-Vereinigung*, vol. XI, p. 343. "Die synthetische Geometrie im Altertum und in der Neuzeit."

The history of the teaching of geometry shows, however, that the step would not be without parallel. In 1570 Sir Henry Saville began to lecture at Oxford upon Greek geometry; in 1619 Briggs first held classes in Euclid at Cambridge. Yale introduced geometry in 1733 and Harvard in 1737. Such has been the career of a subject now deemed proper for the secondary school sophomore.

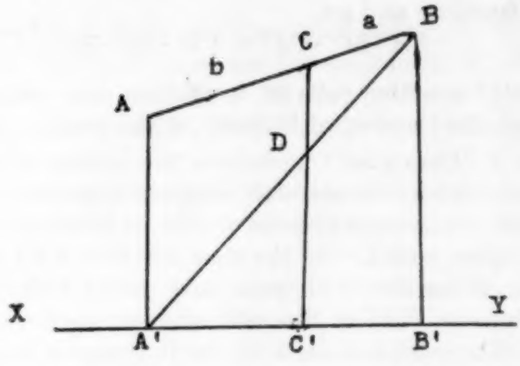
There have been attempts to incorporate the elementary notions of synthetic geometry into the course as classically presented. The general dissatisfaction with the subject matter and methods of teaching Euclid inspired the appearance of a score or more of texts covering a period of about sixty years. It is the purpose of this paper to present certain data of a critical and historical character relative to the movement which it is hoped will be of interest to some of the readers.

The most extensive bibliography to come to the author's attention is the article by Max Simon, "Die Entwicklung der Elementar-Geometrie in XIX Jahrhundert", published in the Jahresbericht der deutscher Mathematiker-Vereinigung, Band I-2 (1906-1908). The article, which is fifty two pages in length, is divided into three parts devoted to history, methods and text books respectively. The list purports to deal with elementary geometry but in view of the great divergence between the schools of Europe and America it is well nigh impossible to determine without actual inspection whether or not a text is of the grade used in the high schools of the United States. The books mentioned in the following paragraphs are all of a nature adaptable to American secondary instruction.

The earliest work appears to be the "Elements de geometrie" published at Liege in 1866 by Eugene Catalan. It is a book of three hundred and sixty pages of which those from 106 to 119 are devoted to the elementary ideas of projective geometry The material comes in the appendix to the third chapter and is introduced by the footnote, "This appendix contains the principal notions of the various theories created by Desargues, Pascal, Carnot, Brianchon, Poncelet and Chasles and constitutes modern geometry."

It is interesting to note the method of attack in this first effort and hence theorems I and III are given in detail.

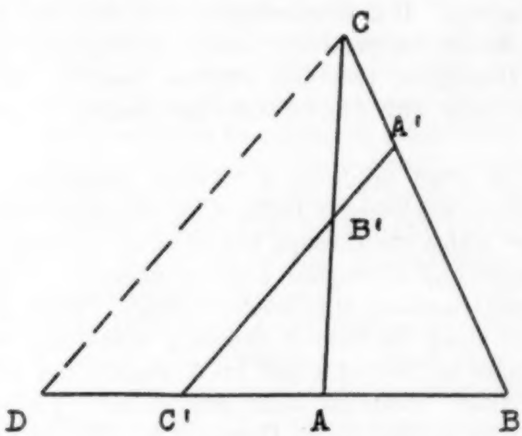
THEOREM I: If a line AB is cut into two segments AC and BC proportional to b and a respectively and if from A, B,



and C perpendiculars AA′, BB′, and CC′ are dropped upon any line XY then it will be so that

$$(a+b)\ CC' = aAA' + bBB'.$$

The proof follows immediately from a consideration of the pairs of similar triangles A′C′D′, A′B′B and A′AB, DCB.



THEOREM III: Any transversal A′B′C′ determines upon the sides of a triangle ABC six segments such that the product of the three non-consecutive segments is equal to the product of the three others.

Proof: Draw CD parallel to A′C′.

$$\text{Then } \frac{DC'}{CB'} = \frac{AC'}{AB'}, \text{ and } \frac{CA'}{DC'} = \frac{BA'}{BC'}.$$

$$\text{Multiply and get, } \frac{DC'}{CB'} \cdot \frac{CA'}{DC'} = \frac{AC'}{AB'} \cdot \frac{BA'}{BC'}.$$

Clear of fractions and get,

$$AB' \cdot BC' \cdot CA' = CB' \cdot AC' \cdot BA'.$$

This result the author calls an involution and remarks that the converse, the theorem of Ptolemy, is also true.

Theorem V (Desargues') introduces the notion of an axis of perspectivity and by means of it progress is made toward the consideration of harmonic division of a line and the concept of harmonic conjugate points. By the time theorem XIII is reached the student is familiar with poles and polars with respect to circles. Theorem XVI is the celebrated hexagon relationship of Pascal. The section closes with the theorem of Brianchon to the effect that the diagonals of an inscribed hexagon meet in a common point.

The book is well put together from the standpoint of arrangement of material and the presentation is vivid and unencumbered by detail. One objectionable feature which the work shares with others of the same period is that the figures are on plates in the rear. It is interesting to note that this first effort does not consider perspectivity, except incidentally in connection with Desargues' theorem, confines itself to figures in a plane, deals only with finite points and makes no mention of duality.

In 1867 H. Pfaff produced a "Neuere Geometrie," and in 1868 J. Versluys published in Holland his "Beginsels der nieuwere Meetkunde" which ran through five editions. During the same year Cremona and Battaglini were members of a commission to investigate the status of geometry in Italy. Their recommendations were along the lines of sweeping reform and resulted in the appearance in 1869 of a text by A. Sannia and E. d'Ovidio under the title "Elementi della geometria." Following this came works by C. Seidelin in Denmark in 1871, by X. Stoll in Germany in 1872, and in 1874 books by J. Lentheric in France and Luciano Navarro in Spain.

The next decade and a half brought forth in Germany the most ambitious of any of the attempts to place modern geometry in the public school curriculum. It was felt that the great work being done by Moebius, Steiner and von Staudt should be recognized and appropriated in so far as possible to the secondary schools. The result of the sentiment was a series of books by Schlegel and Fiedler, Muller, Kruse, Becker, Worpitzky, and Henrici and Treutlein and in addition lectures and manuals

for teachers by M. Pach and Freiderich Meyer.

The most worthy of these was the work in three volumes published at Leipzig in 1881 by J. Henrici, professor in the gymnasium at Heidelberg and P. Treutlein, director of the gymnasium at Karlsruhe.

The authors are not of the opinion that the book should be pursued rigorously in the order written but on the contrary express the hope that the teachers will find opportunity for the utmost freedom in the exercise of their personal judgement.

No attempt is made in the first volume to do more than present the customary theorems of Euclid but in some cases the notion of duality is introduced and also it is made plain to the student that one figure may be obtained from another by means of projection.

The second volume is in two parts, the first being devoted entirely to modern geometry. After treating the subject of proportion and the division of a line in a given ratio the authors enunciate the following on page eleven.

"One figure is called the image of another if the two are so located that the lines joining the corresponding points of the two figures all pass through a common point and if every point of a line of one corresponds to a point of a line of the other. In this position the figures are said to be perspective to each other."

Then follow the theorems of ordinary geometry dealing with lines cut by parallel transversals, lines drawn parallel to a side of a triangle and in fact all of the considerations that might arise from perspective line segments.

On page thirty-two we find the following: "If two similar figures are in perspective position to a third they are also in perspective relation to each other and the three centers of perspectivity lie upon a line called the axis of perspectivity."

The contents of the second section may perhaps be best summarized by the following quotation from the preface.

"In the second section which treats of figures with an axis of projection we have chapter IV which concerns itself with the theorems of Menelaus and Ceva and those derived from them and likewise chapter V which treats harmonic point ranges and pencils of lines, independently of what preceded. Following this comes new versions of the theorems concerning point ranges and pencils of rays in order to lay the foundation for a new treatment of the theorem of Menelaus and the theorem of two double rayed pencils which has the same significance for

the second section as the theorem of the double rayed pencil and parallel lines has for the first. On the other hand the sine relations and involutions are separated."

"If one desires to go rapidly and easily from the perspective properties of the circle to the representation of circles with an axis of projection then the harmonic relations of sections 8, 17-19 may be omitted."

"If one has in view the most direct means of reaching the conic sections via the theorems of point ranges and pencils of lines one may on the one hand go from the theorem of Menelaus to that of Carnot, or on the other hand complete the leading projective theorems of section 22 and in addition those concerning points at infinity and then in the chapter concerning circles advance to the theorems of Pascal and Brianchon."

The third volume begins with the theorem of Desargues and then discusses vanishing lines and vanishing points, quadrilaterals and quadrangles, harmonic division of a line with attention to certain cases of points at infinity, projection of the circle, poles and polars, inscribed and circumscribed quadrilaterals and hexagons, Pascal's theorem, conic sections as projections of the circle, determination of the conic sections by points and tangents, and axes, foci and directrices.

Mention was made of a text by Muller. It was entitled, "Leitfaden der ebene Geometrie mit Benutzung neuer Anschaunungsweisen," was a most excellent book, and received the following critical review.[2]

"To choose from the wealth of material of new geometry is no small task. The work appears throughout to have the proper scheme to arouse and to maintain in assiduous and zealous students an interest in geometry. We recommend the book most warmly to the teacher of mathematics and close with the judgement of Professor Clebsch concerning the same manuscript to the effect that the need of the present time is that the hand of modern geometry play a greater part in the teaching of school geometry and that the aforesaid manual is a most welcome gift to the schools."

The book contains one hundred and thirty seven pages. It makes no mention of involution, harmonic division, poles and polars or of the theorems of Desargues, Pascal or Brianchon. It does treat of points at infinity and handles quite completely

[2] H. S. Shotten, *Inhalt und Methode der planimetrischen Unterrichts*, vol. I, p. 21, footnote (I).

the notions of duality, perspective, and the tracing of the conic sections from the projection of the circle.

In spite of the excellence of this text and also those of Henrici and Treutlein they rarely reached a third edition. It is of interest to note that at the same time the vastly inferior and orthodox geometry of a certain Kambly enjoyed widespread usage and went to one hundred editions. Max Simon compares it with the meteoric career of Wentworth's geometry and observes that, "it was an unparalleled success in spite of, or possibly because of, its entire lack of logical value."[3]

The flame appears to have been extinguished in Germany after 1890 by the deluge of Kambly editions. In Sweden, however, we find a text by P. G. Laurin, in Spain one by Jose Echegaray and in England works by John Casey and J. A. Third. None of these volumes has come to the author's hand and it is impossible to state whether or not they are all of an elementary nature.

The best and by far the most complete and pretentious attempt to combine ancient and modern geometry is the two volume work of Rouché and Camberousse. That this effort was a success is evidenced by the fact that the edition of 1900 was the seventh. In the preface the authors specifically state that in order to apply a science one cannot be restricted to certain portions of it. Claim is made that modern geometry having been deleted from the school curriculum by the "programmes officiales," its great discoveries have not been accorded a deserved place in the science of mathematics. Consequently the authors have devoted some ninety pages to projective geometry and note as evidence that the scheme is a proper one the fact that the demand has exhausted seven editions.

The section devoted to modern geometry begins with a discussion of directed line segments and of positive and negative arcs. Next come the theorems concerning the transversals of a triangle beginning with those of Menelaus and Ceva. The material on through to the end is exhaustively treated both with respect to the plane and to space and embraces everything mentioned in all the other texts and in fact much more because it goes into the fundamental notions of surfaces. It is a text which one might take up with no knowledge of plane geometry but which if pursued would prepare one for metric and differential geometry.

---

[3] *Jahresbericht der deutscher Mathematiker-vereinigung*, Band I-2, p. 33, line 31 and p. 42, line 14.

In recent years in England there have been several texts among which are "Plane Geometry for Advanced Students," a two volume work by Durell and an "Elementary Treatise on Pure Geometry" by John W. Russell. While these are excellent things of their kind they are hardly adaptable to the high school as at present organized and operating in this country.

The only simon pure attempt to incorporate the essentials of projective geometry into a regulation "plane geometry" in the United States is, so far as the author knows, the book entitled, "Elements of Geometry" which appeared in 1897 written by Professors Phillips and Fisher of Yale University. Thirty-six pages at the very end of the book give the presentation of one hundred and thirteen theorems and exercises beginning with the transversals of triangles and treating quadrilaterals and quadrangles, properties of circles, linkages, stereographic projection, poles and polars, the nine point circle, perspective, duality, involution and pseudo-spherical geometry.

For the college undergraduate there have been books written by Dowling, Miss Scott and Lehmer. In the preface of his text Professor Lehmer expresses the belief that the subject will eventually force its way into the secondary curriculum and hopes that his little book will aid in the movement.

The advisability of introducing modern geometry into the secondary schools is no part of this article. It might be noted in closing, however, that a study of circulation statistics appears to lead to no conclusion. The French text of Rouché and Camberousse went to seven editions while on the other hand those of Catalan, Henrici and Treutlein, and Muller enjoyed at most three reprints. Just what would have happened had not the German books been in competition with the Kambly geometry is impossible to state. The outstanding fact of the whole investigation is that the experiment of introducing modern geometry into the secondary schools has been tried with varying success which in no case has been of a lasting nature.

LOYOLA EDUCATIONAL INDEX.

This is the name of a new educational publication edited by Dr. Austin G. Schmidt, Dean of the Graduate School of Loyola University. It appears five times a year, subscription price $15.00 per year. It is a subject and author index to current publications in education and psychology and lists all the leading educational and psychological journals. Address, Loyola Educational Index, 3441 North Ashland Ave., Chicago.

THE BIOLOGY COURSE OUTLINED IN MAJOR OBJECTIVES.

BY ELLIOT R. DOWNING,
The University of Chicago.

Biology, at secondary school level, is a means to an end, not an end in itself. The subject may be, to the professional biologist, its own justification for his devoted study, but certainly to the high school pupil this is rarely true. Biology at the high school level is a cultural, not a vocational subject.

Teaching is a process, the success of which is to be measured in terms of the changes produced in the student. The objectives to be achieved in teaching biology must be stated in terms of such desirable alterations in the pupil if the biology teacher is to consciously achieve worth while outcomes. Then with these definite goals in mind, that subject matter will be selected, those methods of instruction chosen which will best produce the effects wanted.

To state these aims of biology instruction in terms of recapitulation is particularly satisfactory to the biologist. Education is to him the process of bringing up the little animal we call a baby as rapidly as possible through savagery and dawning civilization to this twentieth century. That individual is cultured who has ingested, digested and made a part of his own being the finest fruits of racial experience. We can not give the pupil all the scientific racial experiences but we can hope, by the biological instruction in the schools, to produce in the pupils those most significant effects which biological contacts have contributed to racial advancement.

What, then, are the major contributions which in retrospect we can see biology has made to mankind's progress? They may be classed as:

I. New skills based on a knowledge of biological laws and principles such as skill in healthful living, skill in producing better plants and animals to serve man's needs.

II. New emotionalized standards, ideals and tastes that have been engendered by the self-sacrificing devotion of scientists, by new appreciations of the significance of the biological environment.

III. That new attitude of mind we call scientific, for

which not biology alone, but all science is in large measure responsible, but to which biology has made striking contributions in the work of such men as Jenner, Lister, Koch, Pasteur, Manson, Reed, and a host of others. These men have worked along lines that conspicuously affect every man's life and so challenge his attention.

The course in biology should be outlined first in terms of these objectives not in terms of the subject matter to be taught. The selection of the subject matter best suited to reach these really important goals is the second step, not the first.

Many present day texts in biology have been prepared by professional biologists who are so impressed with the importance of the subject that they endeavor to present a survey of the whole field to the secondary school pupil. Instead, a few things need to be so taught that they are ready for efficient use as the pupil faces the biological problems of his life, not those problems alone that concern earning a living but those primarily that are involved in living a life. The present course in biology is more often a curiosity shop than a tool chest.

Specifically then what are those laws and principles of biology an understanding of which is *most* needed by the average man to enable him to successfully adjust himself to his environment? What are the *most* important emotionalized standards, new tastes and ideals, that biology can help establish in its students? What are the elements that constitute the scientific attitude of mind most commonly lacking in pupils on which consequently they *most* need drill?

Note the emphasis on the word most. The biology course should consist not only of what is good but of what is best. The time is short; to fritter it away on inconsequentials is criminal.

Unfortunately the answers to the questions proposed can not be given on any authoritative basis. There is needed a series of investigations to determine the biological knowledge of most importance. Finley and Caldwell[1] made a start in their study of what a man needs to know of biology

[1]Finley, C. W. and Caldwell, Otis, W., *Biology in the Public Press*, Lincoln School, Columbia University Bulletin, 1923.

to read the articles dealing with the subject in the daily press. Their report is in terms of important topics, not the significant principles. These most important items are in order: Health, especially the germ nature of disease, the Improvement of Animals and Plants, the Cultivation of Plants and Food Costs.

Paul L. Palmer[2] in a study of the material in the Literary Digest, has shown that Health is the topic of major importance. Next comes the Development and Conservation of Resources, then Mechanics and Invention, etc. Again the results are in terms of topics, not principles.

Miss Arlee Nuser[3] has given a list of the most essential laws, principles and concepts needed by the farmer to comprehend the articles on chemistry in his trade journals. Similar studies are needed for biology covering not only farming but other lines of human activity. Miss Beulah Coon[4] has made a study of the important laws and principles of science needed as a basis for understanding the instruction in domestic science and home economics.

These studies are cited as types of the many that will be needed to determine, on a fact basis, what are the really important laws, principles and concepts to be imparted in science instruction. When that is done, the teacher of biology at secondary level must find out experimentally which ones can most successfully be put across at the high school age.

Similar studies must also be made to determine the most significant emotionalized standards to be established in pupils, the most important elements in the scientific attitude of mind, and at what school level each can best be imparted.

But we can not wait for all these things to be done. A biology course must be formulated and taught now. The teacher must judge for himself what are the important things to be included in the course for his community. He probably will not go very far astray if he is reasonably familiar with the history of biology so he can evaluate its

---

[2]Bobbitt, Franklin and others, *Curriculum Investigations*, Supplementary Educational Monographs, No. 31, The University of Chicago.

[3]Master's thesis, The University of Chicago, The School of Education, 1925.

[4]*Suggestions for Content and Methods for Course in Science Related to the Home*. Federal Bd. Vocational Ed. Washington, D. C.

great contributions to man's welfare, and if he will block out his course in terms of these major goals.

To make this concrete there follows an outline of such a course. This is not presented as a stereotyped course to be followed country over but is merely suggestive of the way in which the course should be blocked out in terms of the important objectives as outlined above.

I. Knowledge in Terms of Principles and Laws.

Unit 1. To make clear the principle of the germ nature of disease and to give much drill in the application of this principle to problematic life situations. This will involve the clarification of concepts necessary to an understanding of the statement of the principle. What are bacteria? Accurate notions must be given of their minute size, abundance, rate of reproduction, etc., not merely in words but in experiences. When the principle is grasped it is applied to the explanation of such questions as: Why do we wash our hands before meals? Why isolate a child sick with scarlet fever? Why have contagious disease hospitals? Why boil the baby's bottle? It is only when pupils are led to think through a good many such problems for themselves that they will likely apply the principle in life situations where it is needed—apply an antiseptic to a scratch, cover the sneeze with a handkerchief, wipe up the floors, especially when baby is creeping, with a sterilizing solution. It is impossible to have the pupil learn just what to do in all instances for they are numerous and varied; if he understands the principle and has a good deal of drill in its application to some concrete problems, he will likely recall it and apply it to others when they arise.

Unit 2. To give an understanding of the law of the conservation of energy as it applies to the human machine in the selection of foods on a basis of their calorie value. Again one must take time to clarify by appropriate experiences the meaning of energy, kinetic and potential, the transformation of energy from one sort to another, some methods and units for the measure of energy, how the calorie value of food is determined outside the body, the nature of burning and how it goes on in the body. Much knowledge must be acquired as a background for a real

understanding of the law. Then again drill must be given in the application of the law to the wise selection of foods, so that the child will probably apply it even if he goes to distant regions where the particular foods are strange.

Unit 3. To lead to a comprehension of the storage of the energy of the sunlight in plants and animals with a view to making clear the essential elements in plant cultivation. Only such absolutely necessary items of plant structure and function will be taught by vivid pupil experiences as will give the background needed to comprehend the principle. Then again much drill must be given in solving problematic situations on the basis of the knowledge acquired. Why does girdling a tree kill it? Why are potato sprouts in the cellar white? Why do you eat the leaves of a cabbage, but the root of a turnip and the seeds of the bean? With abundant drill the pupil will, it is hoped, recall and apply the principle when needed in some problems that arise, say, in his own back yard garden.

Unit 4. To teach Mendel's laws so effectively that pupils will comprehend the principles underlying the improvement of plant, animal and human stock. That involves giving many preliminary experiences to pupils to insure that the words used in stating the law and its discussion shall have meaningful content. When the law is understood, facility must be given in its application to problematic situations until the pupils can use it. Then one may reasonably expect that they will on their own initiative buy blooded stock, on the basis of ancestral performance, improve their own plants and animals on the farm skillfully, think of their own matings in terms of the character of their offspring and have a better understanding of some of the complex social problems now facing us that only skill based on the education of the whole body of democracy can solve.

The pupil skills that will ensue from these four units may not be, in the reader's estimation, the most important ones. They are not presented necessarily as such but as types of the outcomes that are worth while.

Note that the thing aimed at in each case is the acquisition of a skill that functions in real life situations of vital importance to the pupil. The knowledge is acquired not for its own sake but as the basis for the desirable skill and the

efficiency of the instruction is measured in terms of the ability of the pupil to apply his knowledge to the solution of the problems he meets. Only when knowledge is in the form of principles is it most readily applicable.

II. Emotionalized Standards.

Unit 5. A study of the work of Pasteur, Koch, Jenner, Lister and others in developing the idea of the germ nature of disease and its application to the problems of human welfare. The purpose of the unit is not primarily knowledge but to awaken in the pupil admiration for the heroic self-sacrifice of these men in the interests of human welfare. Readings of selected passages in the lives of such biologists, reports by pupils, discussion, possibly a biographical sketch presented by the teacher, if he can so saturate himself with the life and spirit of some one of them that he can impart their contagious enthusiasm without being "preachy," may all be means to the desired end.

The results to be achieved can not be tested by the usual written examination. They must largely be taken on faith. Perhaps some line can be obtained on the outcomes by noting the sorts of books pupils are drawing and reading voluntarily from the school or public library. The child's family may see the effects of such instruction more readily than the teacher, or the results may appear only in later life.

Unit 6. A study of the marvelous adjustment of the organism to its environment and of the environment to the organism. The purpose of the unit is to produce a feeling of wonder, of awe and reverence for the world in which the pupil lives—a revelation directly to him quite as potent as that which came through ancient seer and prophet. Let the teacher read Thompson's "The Wonder of Life" and Henderson's "The Adjustment of the Environment" and watch his own changing attitude of mind as he reads. Something of the effects these books will produce upon the teacher is the effect he must try to produce on the pupils. The teacher must select for the purpose, pupil experiences out of their immediate environment and simplify the factual materials presented so as to adapt them to the high school age. The success of the unit is hard to measure.

It can be determined if the facts have been learned but whether or not they will have generated the desired attitude of mind will depend in no small measure on the intensity of the teacher's own response to stimulus of this kind. Such attitudes of mind are contagious and are *caught* by the pupils rather than *learned.*

III. Skill in Scientific Thinking.

Units 7-10. A series of problems and projects,—and the term project is used to mean a problem in the concrete,—in which the teacher will supervise the pupils thought processes with a view to drilling the students in ways of sound thinking. He will make them aware of those safeguards that must be thrown about the elements of the process to make it truly scientific. He will furthermore correct their errors and strengthen their points of weakness. Incidentally some valuable new knowledge will be acquired but that is not the major goal to be achieved.

One class in which instruction in human hygiene and sanitation was a required part of the biology work undertook to reduce the number of fatal accidents in their community—and accidents are the chief cause of death in the ages from 5-14 years. Another class tackled the process of bringing into renewed bearing some old apple trees that had ceased to produce marketable fruit. Still another tried rearing and caring for several kinds of the showy fish that can be kept in aquaria and are always welcome as ornamental household pets. Unwin's little book "Pond Problems" (Cambridge University Press) suggests a number of problems in connection with aquariam work. One class undertook to collect all the species of snails both land and water in their region with notes on their habitats to see if they could so demonstrate any of the laws of geographical distribution about which they had read.

By way of illustration consider the project of bringing into renewed bearing some old apple trees. When the class visited these trees there was presented a complex situation. It had to be analyzed into its elements, the several component problems recognized and clearly defined. The repeated recognition by pupils that this is true of most problematic situations will help them to pick out the essential

elements in their own perplexing problems.

One boy suggested that the trees should be sprayed, saying in support of his proposal that his father, who owned an orchard, always sprayed his trees. The teacher took the opportunity to ask the class if they thought this boy's reasoning sound. The discussion that followed brought out the danger of reasoning by analogy. The boy's suggestion might raise the question as to whether these trees needed spraying, but it would have to be decided, the class thought, on more adequate facts than merely that other apple trees in good bearing in the neighborhood were regularly sprayed.

The teacher asked the class to watch for other cases of inadequate reasoning of a similar sort. One pupil brought in an advertisement which read somewhat as follows:

Do you have that tired feeling? Are you listless, without your customary energy and pep? Then take—and there followed the name of a familiar patent medicine. There were a couple of testimonials from people who had experienced renewed vigor after taking the remedy suggested. The pupils' comment was to the effect that the tired feeling was probably a complex situation like that of these run down trees. It was not safe to conclude that because somebody thought he had been benefited by a medicine you would also be. You might not have the same trouble as he had. It needs careful expert study to find out what the matter is in each particular case and to prescribe the proper remedy.

The class decided it would be well to find out, by appropriate reading, what conditions indicated the need of a spray and then to see if the trees in their care manifested such conditions. Here was a felt need for information to be acquired by consulting authoritative writers. There followed, when the knowledge was obtained, several visits to the trees during which purposeful observation progressed under the direction of the teacher who now had opportunity to make pupils realize the need of accuracy and adequacy in collecting data. The necessity of a regard for such safeguards of correct thinking were further demonstrated in the process of identifying the pests found.

When the facts of the case were determined the class worked out a cogent statement of the course of reasoning

that led to their conclusion in regard to the need of sprays and the choice of sprays to be applied. In the course of this the teacher forced the pupils to judge the sufficiency and aptness of the data to insure a safe conclusion, to state the argument clearly and to critically see that the conclusions only went as far as the evidence justified.

Then came the task of devising with the supplies at their command an apparatus to apply the spray. This gave free play to pupil ingenuity and imagination, accurate diagramatic representation of the appliances suggested, critical examination of these hypothetical solutions of the problem to judge their practicability, and discussion of the principle involved. Three out of several suggested appliances were apparently about equally promising. The class decided to build the three and test them out experimentally to see which would throw the spray farthest and cover the largest area. Of course the teacher had wisely guided the class into this line of action to make clear to them that this is the scientist's method of testing out rival hypothesis. He had given the history of some rival theories like the corpuscular and wave theory of light and described with demonstrations some of the critical experiments devised to determine which was more probable. So the good teacher will draw from the history of science incidents to illustrate the effective elements of scientific thinking and the safeguards to be thrown around them to make his pupils aware of them and of the pitfalls they are likely to fall into in their use of the scientific method.

A detailed statement of the technique of handling the other projects suggested is not necessary. Enough has been given to make clear that the problem or project method is used primarily to give the teacher opportunity to drill pupils in the use of the scientific method. His concern is with the mental processes of his pupils. Needless to say he must himself be aware of the elements involved in reflective thinking and the safeguards to be thrown around the various steps to make it really scientific. Certainly, if there is any one thing in the child's education for which science instruction should be primarily responsible, it is skill in the scientific method, that results in the attainment of the scientific attitude of mind.

THE DEVELOPMENT OF THE FUNCTION CONCEPT.

By G. A. Miller,
University of Illinois, Urbana, Ill.

Two hundred and fifty years ago there was no technical mathematical term which represented our general concept of function. Now we are emphasizing this concept more and more even in elementary mathematics. This is merely one of the many evidences of great changes even in elementary mathematics during the last two or three centuries. It would, however, be very far from the truth if we assumed that the concept of function had not been used long before a special name for this concept was adopted. One of the clearest illustrations of the fact that a concept may appear in the mathematical literature long before it receives a special name is furnished by the fact that Euclid proved the commutative law of certain multiplications in Book VII of his *Elements,* but no special name for this very fundamental law has yet been found in any writings preceding the article by F. J. Servois in the well known French mathematical journal entitled *Annales de Mathématiques pures et appliquées,* volume 5, 1814, page 93. In this particular case more than two thousand years seem therefore to have elapsed from the time that a fundamental mathematical concept was explicitly used until it received a special name.

While the general notion of function did not receive a special name before the latter part of the seventeenth century various special functions were named much earlier. Prominent among these are the trigonometric functions of an angle. It is well known that a special name for such a function appears already in the celebrated Egyptian work by Ahmes. What is perhaps still more interesting is the fact that the ancient Egyptians used the ratio definition of this function in accord with the most common modern usage. Later the Greeks introduced a geometric definition for another function of an angle such that the value of this function depends not only on the size of the angle but also upon the size of the circle at whose center the vertex of this angle was

supposed to be located. It is probably due to Greek influence that for more than a thousand years the trigonometric functions of an angle were commonly regarded as functions of two variables; viz., the angle and the diameter of the corresponding circle. In particular, John Napier (1550-1617) assumed in his tables that sin 90° = 10,000,000. This sine was known in his day as the *sinus totus,* and was equal to the radius of the circle employed in considering the functions of an angle.

It is not yet determined whether the Greeks used the trigonometric function which is now known as the sine of an angle. In the excellent recent work entitled *Geschichte der Elementar-Mathematik* by J. Tropfke it is stated (volume 5, page 7) that the assumption that sine-trigonometry arose in Alexandria cannot be entirely discarded in view of the fact that the sine function appeared in the noted *Pulisa-Sidhañta.* What is of especial interest in connection with this reference is the fact that it is here stated, as well as in various other places, that the name of this Hindu work discloses that it relates to the Alexandrian scholar Paulus (about 378 A. D.). On the contrary, we find on page 308 of volume 2 of the *History of Mathematics* by D. E. Smith reference to "a certain astronomer Pulisa to whom Brahmagupta refers," and a foot-note—on the same page—in which it is stated that "nothing is known concerning the life of Pulisa." If Tropfke is correct there are quite substantial reasons why nothing is known about such a life. In view of the fact that the *Pulisa-Sidhañta* is of such great importance for the determination of the influence of ancient Greek mathematics on the early development of mathematics in India one is naturally greatly interested in the significance of the word Pulisa.

Six very fruitful concepts of mathematics are represented by the following terms: Number, operational symbol for an unknown, limit of an infinite series, system of postulates, function, and group. The last two of these do not seem to have received special names before the latter part of the seventeenth and the latter part of the eighteenth century respectively, and it has sometimes been assumed that they relate essentially to modern mathe-

matics. While the clarifying and systematizing influence of these concepts cannot be fully appreciated until a considerable body of mathematical facts has been accumulated it is easy later to trace their dominating influence to the very sources of mathematical endeavor.

We have no evidence to support the view that the concept of trigonometric function of an angle found in the work of Ahmes was transmitted to the ancient Greeks. In fact, little is known about the influence of this work on the later development of mathematics. In particular, we do not know whether the rule found therein to express the area of a circle as a function of its diameter was transmitted to the following generations notwithstanding the fact that this rule gives rise to a much closer approximation to the value of π than many others which were used later, and that it appears again in a work of the eleventh century. In the history of mathematics it is obviously very desirable to separate those developments which seem to have had only a temporary and local influence from those which were the source of a continuous later development. This separation is naturally often very difficult and is likely to be affected by later discoveries. It may, however, be said that in the present state of our historical knowledge it cannot be assumed that the notions of functions found in the work of Ahmes influenced the later development of these notions among the Ancient Greeks.

One of the most primitive uses of the concept of function is found in the ancient rule for finding the area of a rectangle by means of the product of two adjacent sides. The concept that the area of a rectangle is a function of two adjacent sides is so elementary and so fundamental that it may be assumed to have been discovered independently by different peoples very early in their cultural progress. The notion of function is therefore one of our earliest mathematical notions, and its earliest history naturally antedates reliable records. Here, as in many other domains of the history of mathematics, one is tempted to engage in conjectural speculations, which are often of the deepest interest but should always be carefully separated from the definitely established historical

facts. Mathematics is concerned mainly with the construction of intellectual highways of thought and these highways were usually preceded by trails where the footsteps of the earliest travelers had become entirely obliterated before the time when permanent records of intellectual advances could be made.

As far as we know now the ancient Greeks were the earliest people who endeavored to give due credit to particular individuals for advances in mathematics. Various legendary accounts relate to earlier efforts along this line, especially in China, but the evidences relating thereto are so contradictory as to make it impossible at present to determine the facts. In particular, the development of the concept of trigonometric functions of an angle can be traced continuously back to the ancient Greeks, and this development presents many elements of unusual interest. Among other things it gave rise to a subject which for a time received considerable attention and was known by the name of *Prosthaphaeresis* at about the time when the logarithmic function began to be used. It is a very singular fact of mathematical history that the subject of Prosthaphaeresis, which was based upon such formulas as

$$\sin A \sin B = \tfrac{1}{2}[\cos (A-B) - \cos (A+B)],$$

received so much attention for a time notwithstanding the fact that the law of exponents offered a much easier method for replacing multiplication and division by addition and subtraction respectively.

The development of the concept of trigonometric function was closely related to the development of the number concept so as to include the negative numbers. In fact, the former development probably influenced the latter more profoundly than is commonly recognized. It is interesting to observe that in the noted *Trigonometry* by Pitiscus, 1600, it is stated that it is impossible for an angle which is larger than 90° to have a secant or a tangent. Since angles which lie in the second quadrant* may appear in a plane triangle it is obvious that the form-

*It may be noted that in the 1927 edition, as well as in many of the earlier editions, of Webster's *New International Dictionary* under the term "trigonometrical" the second and the fourth quadrants are interchanged in the figure used for illustration.

ulas for the solution of the plane triangle must have suffered greatly in generality and elegance as long as negative numbers were avoided in the application of the trigonometric formulas. The usefulness of the negative numbers in the definitions of the trigonometric functions of large angles is so important that the study of these numbers would be justified even if they were not useful elsewhere. Hence much light is thrown on the development of these functions by the fact that no satisfactory explanation of the meaning of the operations with negative numbers beyond addition and subtraction became known until about the beginning of the nineteenth century.

It was noted above that as far as we know now the ancient Greeks were the first to make a serious effort to give due credit to certain individuals for the discovery of mathematical facts. It should, however, be added that such credit was not always correctly bestowed. Gustav Junge published in recent numbers of the *Jahresbericht der Deutschen Mathematiker-Vereinigung* several valuable articles devoted to the history of Greek mathematics. In the first of these he noted (volume 35, 1926, page 67) that the picture of Greek mathematics has been greatly changed during the last half century. Before this time one was inclined to assume that the historical statements found in various places of the rich Greek literature were literally true, and one sought to secure from these historical notes a correct view of the earlier mathematical developments. Now we have found out that this method does not enable us to secure a clear picture of the development of Greek and earlier mathematics.

In the history of mathematics nothing is more misleading than to quote from unreliable sources, since the reader is naturally inclined to assume that such quotations are in accord with the present state of knowledge relating to the questions involved therein. In particular, many of the quotations relating to the discoveries made by Pythagoras are no longer taken seriously by some of the writers on the history of mathematics. This includes those relating to a proof of the fundamental theorem commonly known by his name according to which the hypotenuse of a right triangle is expressed as a function

of the other two sides. Obviously the study of the history of mathematics has become much more difficult since the reliability of various sources upon which earlier writers depended has been questioned, partly because the direct use of these sources often led to contradictions. The scientific progress of the human race has naturally been attended by an increasing emphasis on accuracy, and it is only natural that the earliest accounts of mathematical advances frequently failed to fulfill modern requirements for accuracy. Possibly Euclid was aware of this deficiency in the then current accounts since he did not give a single historical note in his *Elements*, which are now commonly regarded as the most influential textbook ever published.

In these *Elements* a very large number of special functions are considered but these functions are usually expressed in a geometric language. Even the trigonometric function which the ancient Egyptians had expressed in a numerical form was later expressed through the Greek influence in a geometric form. It may be said that through this influence the notion of function was commonly regarded as a geometric notion for more than two thousand years although many steps towards the arithmetization of this and various other mathematical notions were taken much earlier. Even in the arithmetical Books of Euclid's *Elements* all numbers are represented in the diagrams as simple straight lines even when these numbers were the products of two or more factors. In accord with the theoretical character of Euclid's *Elements* the functions used therein were commonly expressed in an implicit form. For instance, we find here that circles are to each other as the squares of their diameters but we do not find the area of a circle expressed as an explicit function of the diameter. Similarly, Euclid proved that the volumes of spheres are to each other as the cubes of their diameters, but at the time of Euclid the Greeks did not know how to express the volume of a sphere as an explicit function of its diameter. This advance was made a little later by Archimedes, who is now commonly regarded as the greatest mathematician of antiquity.

One of the most interesting among the functions considered by the ancient Greeks is now commonly known as *Heron's formula* and expresses the area of a plane triangle as a function of its three sides. In accord with a statement made by an Arabian mathematician various mathematical historians have recently credited Archimedes with the discovery of this formula even if it does not appear in his extant works. In view of the fact that spherical trigonometry received early attention as a result of its usefulness in the study of astronomy it is especially interesting to note here that the functional relation between the area of a general spherical triangle and its side was a comparatively late discovery in the development of elementary mathematics. From the fact that Regiomontanus (1436-1476) proposed to one of his contemporaries the problem of finding the area of a spherical triangle on a sphere whose diameter is 100, when the sides of the triangle are 15°, 24°, 34°, respectively, it is sometimes assumed that he knew already how to express the area of a general spherical triangle as a function of its three sides, but this evidence is obviously not conclusive. At any rate, there is no evidence to show that any one before Regiomontanus could solve a general spherical triangle if we assume that such a solution implies the finding of its area as well as the finding of the unknown parts of this triangle.

The fact that such a useful elementary function as the one which enables us to express the area of a spherical triangle in terms of the spherical excess of its angles does not seem to have appeared in the mathematical literature before the beginning of the seventeenth century, when it appeared in the work of T. Harriot as well as in the well known *Invention nouvelle en l'algebre*, 1629, by A. Girard, throws much light on the development of the function concept. Evidences of slow development must be associated with such evidences of early development as are exhibited by the early appearance of the so-called Heron's formula in order to obtain a true picture of the growth of mathematical insight. In the study of the development of the function concept, as well as of the development of mathematics in general, the beginner is

too apt to confine his attention to the high peaks of attainments. Although these are of the greatest importance they obviously fail to convey the exact situation. While the different facts of mathematics are unusually closely related yet the early developments usually present a very mountaneous aspect where many connecting roads have to be provided by later workers.

The almost endless variety of special functions which had been studied before the close of the seventeenth century naturally called for a more comprehensive view and a clear description of what is common to all of them. This tendency was partially expressed in the works of Fermat, Descartes, and others, which gave rise to the subject of analytic geometry, the ordinate of a plane curve representing the value of the function of the corresponding abscissa. More imperfect steps in the same direction had been taken much earlier by N. Oresme and others. G. W. Leibniz used the term function in 1692 in a geometric sense for a line segment which varied according to a certain law. The same term was used six years later with essentially its modern meaning by the noted Swiss mathematician John Bernoulli I, who gave a definition thereof in 1718. In his *Principia* I. Newton used the term genitae to represent functions. The fact that the need of such a mathematical term was not felt at an earlier date tends to exhibit the type of earlier mathematical thinking and the great popularity of the term function in more recent times bears witness to the rapid progress which has been made during the last two centuries.

One frequently finds the statement that the term function was first used for different powers of an unknown quantity. There seems to be no historical foundation for this affirmation, which was based upon an inaccurate remark made by D' Alembert; *Cf. Encyclopédie des Sciences Mathématiques,* tome 2, volume 1, page 3. From what was stated above it is clear that it is not certain that such powers were the earliest functions considered by the ancient mathematicians. The finding of the area of a rectangular piece of ground is probably at least as primitive as the finding of the area of a square, and

hence the notion of a function of two variables may be at least as old as the notion of a function of a single variable.

In his well known work *Introductio in analysin,* 1748, L. Euler made a systematic classification of elementary functions and introduced, among other things, the fundamental distinction between *uniform* and *multiform functions.* He classified functions purely on the way in which they are formed by means of the independent variables. In particular, he called a function transcendental if it involved the finding of the logarithms, the raising to irrational powers, or integrations giving rise to non-algebraic functions*. When two variables are related by an algebraic equation whose terms are of the form

$$a_{m,n}\, x^m\, y^n$$

where m and n are postive integers or zero, and the coefficients are given constants, then each of these two variables is said to be an *algebraic function* of the other. When y is equal to an expression formed by means of a finite number of sums, differences, products, quotients or roots with intregal indices of a variable x, then y is said to be an *explicit* function of x. By the consideration of *implicit* functions L. Euler made an extension to the notion of function but his definition of transcendental functions was too vague and too incomplete to be maintained. In fact, the only general definition of a transcendental function at present seems to be a negative one, calling a function transcendental in a certain interval if it is not algebraic therein.

During the eighteenth century the study of the problem of vibrating strings led to a generalization of the notion of function and during the first half of the nineteenth century Dirichlet suggested a very general definition to the effect that y is a function of x in a given interval when to each value attributed to x in this interval corresponds a unique and determined value of y, without specifying anything as regards the relation between the various values of y. The generality of this definition

*John Bernoulli used the terms algebraic and transcendental functions at a somewhat earlier date, 1724.

of function is, however, too great for the purpose of deriving by means of it a large body of useful results. It serves, however, to exhibit the very wide range of applications of the concept of function and the tendency towards generalization. Just as J. Hudde (1628-1704) made a great advance by the use of a single symbol to represent both positive and negative numbers, so John Bernoulli also achieved a great advance by the use, in 1694, of a single symbol n to represent a function of the variable x. A very close approach to the modern notation for a function was made in 1718 by John Bernoulli, who employed the symbol ϕx, without enclosing x by a parenthesis as is now commonly done. It may be noted here that the statement relating to the first use of a symbol as given in the *Encyclopédie des Sciences Mathématiques*, tome 2, volume 1, page 6, is not quite accurate, since A. C. Clairaut and L. Euler were not the first to use such a symbol for a function of x as is stated here.

A very great impetus to the study of functions was furnished by the general introduction of complex numbers. While these numbers were used with increasing confidence from about the middle of the sixteenth century and a considerable number of their fundamental properties were developed as early as 1572 in the noted *Algebra* by R. Bombelli*, it was not until about the beginning of the nineteenth century that the legitimacy of their use in the operations of mathematics beyond addition and subtraction was fully established by Caspar Wessel and others. The study of functions of a complex variable was one of the main sources of the enormous mathematical progress realized during the nineteenth century. The discoveries during the eighteenth century of the fundamental relations between the trigonometric functions and the logarithmic and exponential functions contributed powerfully to the general introduction of the complex numbers. The technical term *elementary functions* is now commonly used to include the following: Rational functions, variables whose exponents are rational or irrational numbers, exponential

*Cf. H. Wieleitner, *Jahresbericht der Deutschen Mathematik-Vereinigung*, vol. 36, (1927), p. 74.

functions and their inverses, circular and hyperbolic functions and their inverses, and also the rational functions of these different functions.

The growing emphasis on the concept of function in elementary mathematics is in line with the general development of our subject, where the more advanced branches frequently throw new light on the more elementary ones. It is clear, for instance, that the fundamental operations of addition and multiplication, as well as of their inverses, may be regarded as functional operations since the sum is a function of its addends and the product is a function of its factors. On the other hand, when the beginner adds 5 and 7, or when he multiplies these numbers, he is not apt to view the operations involved from the standpoint of functions. The meanings of such simple operations are, however, greatly enriched when they are viewed as special instances of functional operations. In view of such very elementary possible uses of the concept of function it is obviously very difficult to trace the historical development of this concept. It represents largely a point of view and the record of work seldom exhibits fully what was in the author's mind. The advances made by particular individuals are frequently intangible, where, on the other hand, advances become very apparent when developments separated by long periods of time are compared.

EDWARD JENNER'S BIRTHDAY ANNIVERSARY.

Last month the editor of our Science Questions Department suggested that science classes commemorate the anniversaries of great scientists. This is a good suggestion. May 17th is the birthday anniversary of Edward Jenner, the hero of the war against smallpox. Why not devote a class period or science club program to the work of this great scientist? It is a good opportunity to teach health science in the most effective manner. In addition to the usual sources of information one of the little pamphlets in the Health Heroes series prepared by the Metropolitan Life Insurance Co., will be found valuable. Also Vol. 1, No. 1 of Medical Progress, a new quarterly journal for laymen, published by The American Association for Medical Progress, 370 Seventh Avenue, New York, and edited by Dr. Benjamin C. Gruenberg, gives some interesting items on the results of vaccination.

MOTION PICTURE OR FILM SLIDE?

By H. E. Brown,
Ridgewood High School, Ridgewood, N. J.

Introduction.

In the two investigations reported an attempt has been made to answer the question raised in the title of this article.

At the time that the first investigation was undertaken, a second one was not contemplated until, through the failure of a number of the High School Freshmen to be tested to report, the numbers used were reduced to only 32, or 16 matched pairs, and also through the inconclusiveness of the data, the necessity for a second investigation became apparent.

This second experiment was conducted some two months later, with much larger groups—80 in all or 40 matched pairs, and with a much larger number of test items than in the first attempt.

Both reports were written up individually and no attempt has been made to fuse the results in one common report, as it was felt that the reactions to the data, recorded at the time the investigation was made would be more valuable than a fusion of reactions recorded some time after the completion of both experiments.

Report of First Investigation

Motion pictures and film slides (also called strip film) are so new in the field of visual education, that little material, throwing light on the perplexing question of which is the better method to employ in a given situation, is available. Particularly is this true when the case under discussion possesses elements that are recognized as being best adapted to one method in some cases, and to the second method in other cases. Thus in studying the act of hearing, we attempt to teach the motion of the bones and fluid in that process—surely best presented by motion pictures, according to the rule developed in what is probably the best book on the subject (H. N. Freeman, Visual Education); and then also attempt to show the bones of the ear when at rest—just as surely best presented by a still picture. It was because of this possession of different elements of visual pedagogy and because of the existence of good motion pictures and strip film on this subject, that the subject "How We Hear" appealed to me as good testing material.

At the start be it admitted that the test as employed is not conclusive. The number of cases (32 in all) is not sufficient to use as a basis for any sweeping generalizations. Whatever of value there is in the investigation, lies in the method, which is capable of application to further work along this line.

The objection to most experiments attempting to settle the question is that two important variables are likely to be overlooked. The second of these two variables, in almost all cases that have come to my attention, has been completely disregarded. These factors are:

1. The lack of intelligence uniformity of the groups tested.
2. The inequality of the motion picture and the strip film employed. Thus no investigation, attempting to demonstrate the relative merits of these two methods, can be really valid, unless this second variable item is completely equated in the two cases by using a strip film that contains the same pictures, used as "stills" as are shown in the motion picture, and of a sufficient number to adequately present the subject. Up to the present time this has not been capable of accomplishment unless both motion picture and film slide are made by the same company.

Two groups of High School freshmen were used in this experiment, which was conducted on February 24, 1927. These groups, of 16 each, were selected on the basis of their intelligence as determined by the Otis Group Intelligence Intermediate Test, Form B, administered in June, 1926, at the end of the grammar school careers of these people. The groups were made up by taking two people, whose I. Q.'s were the same or very nearly so, and assigning one to each group. Thus each group possessed the same intelligence range and each person was unconsciously competing with a person of the same intelligence level in the other group.

When the first group had assembled it was explained to them that they had been called together for the purpose of assisting in the carrying out of an educational experiment; that the results had no bearing on their school grades and that they were to be shown a series of pictures on "How We Hear" ("How We Hear" is a film slide put out by the Bray Screen Products of New York City). They were further advised that the pictures would be discussed as shown and that they were to feel perfectly free to ask questions at any time. The strip film was projected by an assistant on a translucent screen in a moderately

dark room and pictures were re-shown as often as seemed desirable. The total time consumed in the showing of the pictures and the discussion was about 20 minutes. This group was then tested as described later. After being cautioned not to discuss the pictures with other freshmen, they were dismissed.

Later in the same day the second group assembled. They were told the purpose of the meeting as in the first group and then the motion picture "How We Hear" (also put out by the Bray Co.) was shown. During the projection, without interrupting the showing, the following questions were asked:

1. "What is this coming up?" (Asked when the picture showed sound waves approaching the ear.) The question was generally answered by the group.

2. "What do these represent?" (Asked at the time when the picture showed the inner ear drum being vibrated and, in turn, varying the pressure on the liquid in the inner ear.) Answered by group, "Variations in pressure."

3. "What do these represent?" (Asked shortly after question 2 above, at a time when the variations in pressure were causing the numerous hair-like strings in the ear to send out a nerve impression to the brain.) No answer from group and answer finally supplied by myself.

After the showing of the picture, which took about 7 or 8 minutes, the same test as for the previous group was given. In each case a mimeographed copy was placed in the hands of each student and they were instructed to write answers in blanks provided. In both groups there was no limit put upon the time that could be spent in completing the questions. In the questions following, it will be noted that eleven items of information were asked.

TEST ON "HOW WE HEAR."

1. The ear is divided into..................main parts.
2. The outer ear acts in a way similar to the..................
3. Sound waves, entering the ear, first strike the.................. causing it to vibrate.
4. This vibration is passed on, first to the.................., secondly to the ...., and next to the..................
5. This last causes the..................to move back and forth.
6. The variation in pressure is passed on by means of a.................. which is in a chamber composed largely of coiled tubes.
7. This variation in pressure causes a number of.................. to vibrate which sends out nerve impulses to the..................
8. These nerve impulses cause the sensation of..................

In the original mimeograph a double spacing was allowed between lines and generous spaces for writing in missing words

were allowed. Answers which were deemed acceptable were as follows:

1. 3.
2. Telephone or telephone transmitter (*not* telephone disk).
3. Drum, or ear drum.
4. Hammer; anvil; stirrup.
5. Inner drum.
6. Fluid or liquid.
7. Hairs, strings or hair-like strings (not things or nerves); brain
8. Sound or hearing.

In tabulating the results in the following table, the letter A signifies that the student whose scores appear on that line was one of the motion-picture group; similarly, the letter B signifies a student in the film slide group. No attempt at percentages was made in the tabulation but simply the number right on the test was recorded. In the last columns are given the advantages of method evidenced in favor of a particular method for the pair of students being compared.

Conclusion.

As stated at the beginning of this article, the cases used, 16 in each group are too few to draw any very general conclusions from. Certain things are rather interesting and may represent a tendency which could be proved by the use of a greater number of cases. Of the sixteen pairs, three are equal, and of the other thirteen, five cases only show a superiority for the motion picture method. And yet these five cases pile up and advantage of 19 as against the 15 which the 8 cases showing a superiority for the strip film method can produce. Is there anything of significance in the fact that, in most instances where the motion picture shows an advantage, it is a large one? May it be that there are "movie-minded" as against "film-slide minded" people? that is people who learn through *motion* better than through *still* pictures? It is hoped that these questions may be answered by a really thorough investigation of this matter.

As I said at the beginning, whatever of value this little investigation may have, is in the pointing out of one way by the use of which, light may be thrown on this question.

Report of Second Investigation

It is evident from a consideration of the results of this first experiment that nothing definite was proved. With the idea of possibly arriving at some definite conclusion with respect to this question the investigation was repeated—this time with two groups of Sophomores whose I. Q.'s based on the Otis

TABULATION OF RESULTS IN TESTS.

	Intelligence		No. Right		Advantage for Method.	
Pupil No.	I. Q.	Score	M. P.	F. S.	M. P.	F. S.
A 1	.89	42	6	....	....	....
B 1	.87	41	....	8	....	2
A 2	.92	42	6	....	....	....
B 2	.92	44	....	8	....	2
A 3	.93	41	4	....	....	....
B 3	.93	47	....	5	....	1
A 4	.94	50	9	....	....	....
B 4	.96	46	....	4	5	....
A 5	1.00	49	5	....	....	....
B 5	1.00	48	....	4	1	....
A 6	1.01	51	5	....	....	....
B 6	1.00	57	....	8	....	3
A 7	1.02	55	9	....	....	....
B 7	1.03	51	....	4	5	....
A 8	1.04	52	4	....	....	....
B 8	1.04	55	....	6	....	2
A 9	1.05	53	10	....	....	....
B 9	1.05	53	....	11	....	1
A 10	1.09	60	9	....	....	....
B 10	1.07	58	....	10	....	1
A 11	1.13	62	9	....	....	....
B 11	1.11	62	....	9	....	....
A 12	1.17	63	9	....	....	....
B 12	1.17	63	....	9	....	....
A 13	1.19	64	11	....	....	....
B 13	1.21	65	....	5	6	....
A 14	1.23	69	11	....	....	....
B 14	1.23	70	....	11	....	....
A 15	1.24	68	6	....	....	....
B 15	1.24	68	....	9	....	3
A 16	1.24	69	11	....	....	....
B 16	1.24	71	....	9	2	....

Total of 5 advantages for motion picture method........19
Total of 8 advantages for film slide........15

Net advantage for motion picture........4

Intelligence test ran through a somewhat larger range of values. The groups also were decidedly larger than with the first in which there were only sixteen in each group.

The procedure was the same in this test as in the former. To the first group was given the strip film, "How's Your Eye-Sight?" put out by the Bray Screen Products Company, New York City, as before. Discussion was free, questions were asked both by the examiner and by the people being tested. The total time required to present the strip film and give the tests was a full period of forty-five minutes. About a week later the second group, whose range of I. Q.'s were the same as the first, were shown a part of the motion picture "How We See" put out by the same company. The strip film is taken directly from this motion picture, but the first part of the motion picture includes items not covered by the strip film, therefore the motion picture was started at a point identical with the first views in the strip film. The only statements made during the course of the picture were:

1. "Notice the shape of the lens." (Made at first showing of a lens to correct an eye defect.)

2. "Notice the movement of the muscle." (This statement made during the showing of eye balls being moved by the muscles.)

3. "What do we call the defect caused by this muscle becoming permanently tight?" (This question asked when motion picture showed a muscle of an eye that was permanently tight.)

The total time for the motion picture and questions was about forty minutes, five minutes less than with the strip film.

One of the faults of the first experiment was the fact that there were not enough questions. This time I administered two separate tests, one of them consisting of fifteen multiple choice questions and the other fourteen false and true statements, both sets covering the same ground. In each case the multiple choice questions were given first, then collected and the false and true administered next. The tests which were given were as follows:

Test No. 1.

Underscore the choice you consider correct, thus in the sample below we underscore *mines* which is the correct answer. Asbestos is obtained from (a) trees (b) *mines* (c) the ocean.

1. The lens of the eye changes shape in order to (A) focus on the object (B) prevent the eye from becoming crosseyed (C) cause the eye to become more beautiful.

2. When the eye is out of focus the image is (A) clear (B) indistinct (C) clear or indistinct.
3. When the eye is in focus the image forms (A) in front of the retina (B) on the retina (C) behind the retina.
4. A flattened eye-ball produces (A) short sightedness (B) far-sightedness (C) crossed-eyes.
5. In short-sightedness the image is focused (A) in front of the retina (B) on the retina (C) behind the retina.
6. A person who is far-sighted can see best an object (A) quite close to the eye (B) at normal distance for reading (C) at long distance.
7. To correct short-sightedness an eye-glass is used that will (A) scatter the rays (B) focus the rays (C) magnify the object seen.
8. The lens which will do this is (A) thicker at the middle than at the edges (B) thinner at the middle than at the edges (C) curved but of equal thickness throughout.
9. A lengthened eye-ball produces (A) short-sightedness (B) far-sightedness (C) crossed eyes.
10. The movement of the eyeball is controlled by (A) one muscle for each eye (B) two muscles for each eye (C) four muscles for each eye.
11. In turning the eyes the muscle on the side toward which it is turned (A) becomes thicker (B) becomes thinner (C) remains unchanged.
12. In far-sightedness the image is focused (A) in front of the retina (B) on the retina (C) behind the retina.
13. A person who is short-sighted can see best an object (A) quite close to the eye (B) at normal distance (C) at long distance.
14. To correct far-sightedness, an eye-glass is used that will (A) scatter the rays (B) focus the rays (C) magnify the object seen.
15. The lens which will do this is (A) thicker at the middle than at the edges (B) thinner at the middle than at the edges (C) curved but of equal thickness throughout.

TEST NO. 2.

Place a + in front of the statements you consider correct, and a — sign in front of those you consider incorrect.

1. Short-sightedness is produced by a flattened eyeball.
2. Far-sightedness is produced by a lengthened eyeball.
3. In order to focus the eye, the lens changes in shape.
4. When the eye is out of focus the image is indistinct.
5. The muscle which turns the eyball becomes thicker on the side opposite the side toward which it is turned.
6. A lens which is thicker at the middle than at the edge is used to correct far-sightedness.
7. Such a lens will scatter the rays of light.
8. One muscle on each eye controls the movement of the eyeball.
9. A person who is short-sighted can best see a distant object.
10. When the image is continually focused in front of the retina the person is said to be short-sighted.
11. When the eye is in focus, the image focuses on the retina.
12. To correct short-sightedness, a lens is used that will scatter the rays of light.
13. A far-sighted person can see best an object close to the eye.
14. If a muscle controlling the movement of the eyeball becomes tight, the person is cross-eyed.

As in the previous investigation in the tabulation of results the letter A signifies that the student whose score appears in that line was one of the motion picture group; letter (B), that he was of the film slide group. The results obtained in each test are given together with the total, and the advantages of either method in the particular case tabulated last.

TABULATION OF RESULTS IN TESTS.

Pupil	I. Q.	Mult. Ch. Test		False and True		Totals		Advantage	
		M. P.	F. S.	M. P.	F. S.	M. P.	F. S.	M. P.	F. S.
A— 1	137	10	....	12	....	22	....	....	....
B— 1	135	....	15	....	14	....	29	....	7
A— 2	135	13	....	10	....	23	....	....	....
B— 2	132	....	11	....	12	....	23	....	....
A— 3	131	13	....	12	....	25	....	5	....
B— 3	131	....	12	....	8	....	20	....	....
A— 4	129	15	....	14	....	29	....	2	....
B— 4	128	....	13	....	14	....	27	....	....
A— 5	126	12	....	10	....	22	....	....	....
B— 5	127	....	14	....	12	....	26	....	4
A— 6	126	14	....	10	....	24	....	....	....
B— 6	126	....	13	....	12	....	25	....	1
A— 7	125	11	....	10	....	21	....	....	....
B— 7	125	....	14	....	14	....	28	....	7
A— 8	123	10	....	8	....	18	....	2	....
B— 8	124	....	12	....	4	....	16	....	....
A— 9	122	12	....	8	....	20	....	....	....
B— 9	123	....	14	....	14	....	28	....	8
A—10	121	9	....	8	....	17	....	....	....
B—10	122	12	12	....	14	....	26	....	9
A—11	120	12	....	12	....	24	....	....	....
B—11	121	....	14	....	12	....	26	....	2
A—12	120	10	....	12	....	22	....	....	....
B—12	120	....	14	....	10	....	24	....	2
A—13	119	11	....	8	....	19	....	....	....
B—13	119	....	13	....	6	....	19	....	....
A—14	118	14	....	6	....	20	....	....	....
B—14	118	....	12	....	12	....	24	....	4
A—15	118	13	....	12	....	25	....	7	....
B—15	117	....	12	....	6	....	18	....	....
A—16	117	9	....	8	....	17	....	....	....
B—16	116	....	14	....	14	....	28	....	11
A—17	116	14	....	10	....	24	....	....	....
B—17	115	....	15	....	12	....	27	....	3
A—18	116	9	....	6	....	15	....	....	....
B—18	114	....	12	....	7	....	19	....	4
A—19	114	11	....	6	....	17	....	....	....
B—19	113	....	14	....	10	....	24	....	7
A—20	113	14	....	6	....	20	....	....	....
B—20	113	....	14	....	12	....	26	....	6

TABULATION OF RESULTS IN TESTS.

Pupil	I. Q.	Mult. Ch. Test		False and True		Totals		Advantage	
		M. P.	F. S.	M. P.	F. S.	M. P.	F. S.	M. P.	F. S.
A—21	110	9	....	4	....	13	....	....	....
B—21	110	....	15	....	12	....	27	....	14
A—22	110	12	....	10	....	22	....	....	....
B—22	109	....	12	....	10	....	22	....	....
A—23	108	13	....	10	....	23	....	....	....
B—23	107	....	14	....	10	....	24	....	1
A—24	107	12	....	6	....	18	....	....	....
B—24	106	....	13	....	12	....	25	....	7
A—25	106	14	....	10	....	24	....	....	....
B—25	105	....	13	....	6	....	19	5	....
A—26	104	9	....	12	....	21	....	....	....
B—26	105	....	15	....	12	....	27	....	6
A—27	104	13	....	12	....	25	....	....	....
B—27	104	....	13	....	14	....	27	....	2
A—28	103	12	....	12	....	24	....	....	....
B—28	103	....	14	....	14	....	28	....	4
A—29	102	13	....	12	....	25	....	9	....
B—29	102	....	10	....	6	....	16	....	....
A—30	101	9	....	6	....	15	....	....	....
B—30	102	....	13	....	10	....	23	....	8
A—31	100	13	....	6	....	19	....	....	....
B—31	99	....	12	....	8	....	20	....	1
A—32	99	13	....	10	....	23	....	....	....
B—32	99	....	12	....	14	....	26	....	3
A—33	98	12	....	10	....	22	....	....	....
B—38	98	....	15	....	10	....	25	....	3
A—34	97	14	....	12	....	26	....	1	....
B—34	97	....	13	....	12	....	25	....	....
A—35	96	12	....	12	....	24	....	....	....
B—35	97	....	13	....	14	....	27	....	3
A—36	96	10	....	8	....	18	....	2	....
B—36	96	....	12	....	4	....	16	....	....
A—37	91	11	....	12	....	23	....	2	....
B—37	90	....	13	....	8	....	21	....	....
A—38	90	4	....	—2	....	2	....	....	....
B—38	90	....	13	....	14	....	27	....	25
A—39	88	13	....	10	....	23	....	6	....
B—39	88	....	11	....	6	....	17	....	....
A—40	84	10	....	....	....	10	....	....	....
B—40	82	....	13	....	14	....	27	....	17

27 instances in which the film-slide proved better, gave a total advantage of	169
10 instances in which the motion-picture proved better, gave a total advantage of	41
Superiority of motion picture in points	128
Instances superiority	17

In scoring the tests the usual methods were followed. That is the number right was counted in the multiple choice test and in the false-true test the number wrong was subtracted from the number correct.

Summary.

In addition to any conclusions drawn at the end of the first investigation, we are forced to the conclusion that in this case at least, the film slide, with the greater exchange of comment that it allows, proved the better. The differences were so large that I am inclined to believe that they would reappear on a similar investigation.

There was one feature of the investigation to which I would like to call attention. The nature of the subject presented was such that it was essentially, particularly in the section showing near- and far-sightedness, a matter of "Stills." That is, in thinking of an eye defect, we recall an object that is not in motion. It is admitted that for the presentation of motionless things, the lantern slide or strip film is the better. In order to present the other side of the picture it would be necessary to present in the two ways used here, some topic closely associated with motion, such as for example uniformly accelerated motion. For the strip film could be used the Spencer Lens Company's subject but the motion picture would, at the present stage of the game, be unobtainable.

I feel that these two investigations can claim to have proved that for much of learning associated with still mental pictures the strip film is the movie's superior as a learning aid. Beyond that we can make no claims.

HIGH SCHOOLS ARE MAKING RAPID STRIDES.

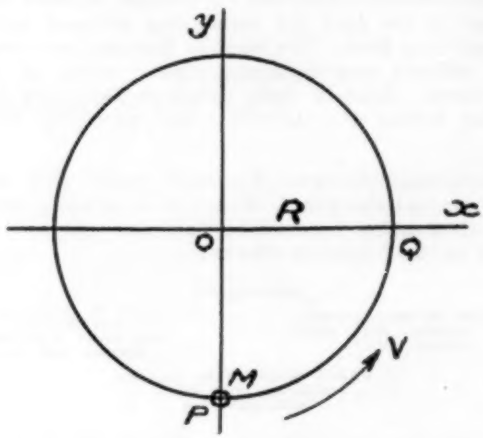
The Bureau of Education has a record of 21,700 public high schools which enrolled 3,757,466 pupils in 1926. The schools enrolling more than 1,000 pupils each number 767, one school enrolling as many as 8,611. These 767 schools represent 3.5 per cent of the total number of schools, and 37.5 per cent of the total enrollment. It takes about 18,000 of the smallest high schools to enroll another 37.5 per cent. One-half of the public high schools enroll fewer than 100 pupils each, the total for these schools being but 13.7 per cent of the whole enrollment.—*Frank M. Phillips in School Life.*

CENTRIFUGAL FORCE.

By F. M. Denton,

University of New Mexico, Albuquerque, N. Mex.

The following is suggested as an alternative to the method usually given in text-books for showing that *Centrifugal force* = MV^2/R. It has the advantage of avoiding the difficulty of thinking of a circle as a many-sided polygon.



M is a mass moving with constant peripheral speed V about the center O of a horizontal circular path of radius R.

A string OM holds M against the constant centrifugal force F acting along the direction of the string.

Inspection of the vector diagrams of the components of the forces and of the velocities along the direction of the line OX while M moves through the quadrant PQ shows that the ratio of the maximum value (F) of the components of F along OX bears to the average value the same relation k, that the maximum value (V) of the components of V along OX bears to its average value. Thus we have $(F/F_a) = k = (V/V_a)$.

Now while M moves from P to Q its average velocity in the direction OX is R/t where t is the time taken; that is to say $V_a = R/t$ or $t = R/V_a$. And the average acceleration (A_a) of M along XO during time t is V/t. Hence the average force along OX during time t is $F_a = MA_a = M(V/t) = MV \div (R/V_a) = MVV_a/R$. But $F = k\,F_a$ and $V_a = V/k$ hence,
$F = k(MVV_a/R) = kMV^2/kR = MV^2/R$.

CENTRAL ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS

The Monthly Message

Our Advertisers. You will be pleased to read that our advertisers of the past are supporting us most loyally for the coming Year Book. We want to increase our commercial friends. Officers and members, please check up on your favorite firms. Send in their names so they may let their light shine before us. Advertise and patronize are reciprocals.

The Geography Section. For some reason this section is due for a revival this year. Where is Geography, and where are Our Geography Teachers? This question will be fully answered by the following officers:

GEOGRAPHY

JAMES C. BAIRD, Chairman
Harrison Technical High School
Chicago, Ill.

HELEN A. SOUTHGATE, Vice-Chairman
Isaac Elston High School
Michigan City, Ind.

MABEL WASHBURN, Secretary
Shortridge High School
Indianapolis, Ind.

Executive Committee Meeting. There will be an important meeting of this committee in the middle or latter part of May. Submit your items of business, bills, or suggestions to the officers in time so they will receive consideration by the committee.

Membership. Our membership committee, led by Miss Weckel, is doing splendid work. New memberships as well as renewals are now coming in regularly. Strange to say, Chicago is reported as slow territory. Come on, Chicago! We expect you to lead.

The Price of Meeting. Some of our eastern educational institutions, such as Harvard and Columbia, make it possible for the members on their faculties to attend distant meetings by providing for a part, or the whole, of their railroad fare. This type of professionalism is as yet too limited, especially in the Middle West. What can we do to convince school systems and higher institutions that our Annual Meeting is worth the price of travel to them? Let us hear from those who have won this consideration.

W. F. ROECKER, President.
Boy's Technical High School, Milwaukee, Wisconsin.

PROBLEM DEPARTMENT.

CONDUCTED BY C. N. MILLS,

Illinois State Normal University.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the problems should be well done in India ink. Problems and solutions will be credited to their authors. Each solution, or proposed problem, sent to the Editor should have the author's name introducing the problem or solution *as on the following pages.*

The Editor of the department desires to serve its readers by making it interesting and helpful to them. Address suggestions and problems to C. N. Mills, Illinois State Normal University, Normal, Ill.

LATE SOLUTION.

996. *Grace Baird, West Bend, Wis.*

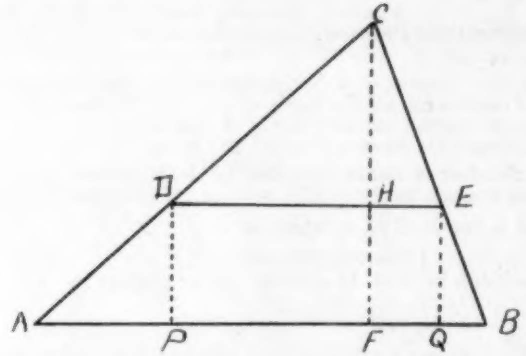
SOLUTIONS OF PROBLEMS.

1001. *Proposed by F. A. Caldwell, St. Paul, Minn.*

Without bisecting any angle, draw a line parallel to the base of a triangle and terminated by its sides so that it shall be equal to the sum of the segments of the sides adjacent to the base.

I. *Solved by R. T. McGregor, Elk Grove, Calif.*

Denote by x the line parallel to AB and equal to AD+BE.



In the figure CF, DP, and EQ are $\perp$ to AB. Then $CH/CF = x/AB$. One readily finds that $HF = (CF - CFx/AB)$; also $AD = AC(AB-x)/AB$ and $EB = BC(AB-x)/AB$. Since $AD+EB = x$ we have

$$\frac{AC(AB-x)}{AB}+\frac{BC(AB-x)}{AB} = x.$$

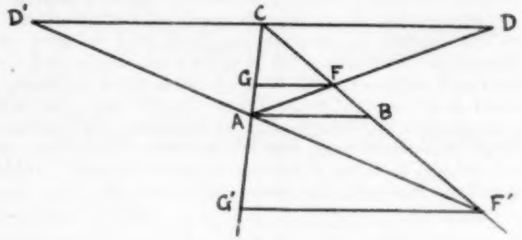
Solving this equation we find

$$x = \frac{(AC+BC)AB}{AC+BC+AB}.$$

DE $= x$ may be constructed as a fourth proportional to (AB+AC+BC), (AC BC), and AB.

II. *Solved by George Sergent, Tampico, Mexico.*

Construction. On the parallel through C to the base AB of the given triangle ABC, lay off on each side $CD = CD' = a+b$. D is within $\angle A$, and D′ within $\angle B$.



Let AD andAD′ intersect the side a in F and F′, respectively. The segments FG and F′G′ of the parallels through F and F′ to BA, between the sides a and b both answer the question.

Proof. The segments determined by FG on the sides a and b are proportional. $AG/AC = BF/BC = (AG+BF)/(a+b)$. (1)
The triangles AGF and ACD are similar. Hence

$$AG/AC = GF/CD = GF/(a+b). \qquad (2)$$

From (1) and (2) we get $GF = AG+BF$.

In a similar manner we find $G'F' = AG'+BF'$.

For the construction, the points G and G′ can be determined by joining B to D′ and D.

Also solved by *E. de la Garza, Brownsville, Texas; J. Murray Barbour, Aurora, N. Y.; P. H. Nygaard, Spokane, Wash,; Smith D. Turner, Cambridge, Mass.;* and the *Proposer.*

1002. *Anonymous.*

What are the chances in a bridge game; (1) that one player shall be dealt a hand containing all the cards of one suit, (2) that each player shall be dealt a hand containing only cards of one suit?

I. *Solved by Smith D. Turner, Cambridge, Mass.*

(1). The number of hands that can be dealt to one player is C52/13. Of these, four contain cards of only one suit. Hence the chances of getting such a hand is one in $C^{52}_{13}/4$, which is

1:158,753,389,900.

(2). The pack can be dealt to a table of four players in

$$C^{52}_{13}C^{39}_{13}C^{26}_{13}C^{13}_{13} \text{ ways.}$$

of these $P^4_4 = 24$ ways will give each player a hand containing cards of only one suit. Hence the chance of this happening is one in

$$(C^{52}_{13}C^{39}_{13}C^{26}_{13})/24, \text{ or about } 1{:}(2235 \times 10^{24}).$$

II. *Solved by J. Murray Barbour, Aurora, N. Y.*

(1). The perfect hand may be in one of four suits. Its cards may be arranged in 13! different ways. The total number of arrangements of any hand is 52! to 13 terms. Therefore the probability is

$$(4 \times 39! \times 13!)/52!$$

(2). Continuing as above for four hands, the probability is

$$4 \times (13!)^3/52!\,.$$

Also solved by *F. A. Butter, Jr., San Jose, Calif.*

1003. *Proposed by M. W. Coultrap, Naperville, Ill.*

Find a number such that its square, when increased or decreased by 5, will remain a square.

Solved by Smith D. Turner, Cambridge, Mass.

If x is the required number, then

$$x^2+5 = a^2 \text{ and } x^2-5 = b^2. \qquad (1)$$

Let c be the least common denominator of x, a, and b; also let $X = cx$, $A = ca$, $B = cb$. Then

$$X^2+5c^2 = A^2 \text{ and } X^2-5c^2 = B^2. \qquad (2)$$

From the identity

$$m^2+n^2\pm 2mn = (m\pm n)^2,$$

and by letting $m = p^2-q^2$, $n = 2pq$, we have

$$(p^2+q^2)^2\pm 4pq\ (p^2-q^2) = (m\pm n)^2. \qquad (3)$$

Identifying (3) and (2), $X = p^2+q^2$, $4pq(p^2-q^2) = 5c^2$. (4)

If $p = 5$, then $4q(p^2-q^2)$ is a square if q, $(p+q)$, and $(p-q)$ are squares. (5)

Since $p=5$, $(q)+(p-q)=5$, and this condition satisfies the conditions of (5) only when

$$\begin{matrix} q = 1 \\ p-q = 4 \end{matrix} \quad (6), \text{ or } \quad \begin{matrix} q = 4 \\ p-q = 1 \end{matrix} \quad (7)$$

(6) yields $p+q = 6$ which violates (5); (7) yields $p+q = 9$. Hence $p = 5$ and $q = 4$ is the only solution in integers for (5).

From (4) $5c^2 = 720$, hence $c = 12$. Therefore $X = 41$, and $x = 41/12$ the required number.

Also solved by *E. de la Garza, Brownsville, Texas.*

Two incorrect solutions were received.

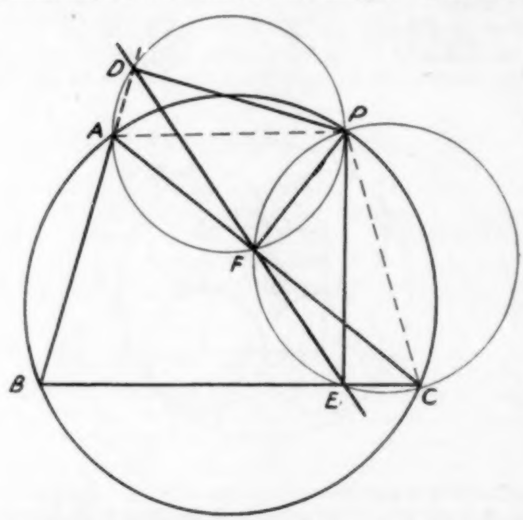
1005. *Proposed by Marquess Wallace, Mexico, Mo.*

From any point on the circumcircle of a triangle, perpendiculars are drawn to the sides of the triangle. Prove that the feet of the perpendiculars are in a straight line.

Editor. This is called the *Simpson line of a triangle.*

Solved by Smith D. Turner, Cambridge, Mass.

Given triangle ABC; point P is on the circumscribed circle. PD, PE, PE are perpendiculars to the sides of the triangle. Draw DF and FE.



Since the lines from P are perpendiculars, circles on AP and CD as diameters will pass through D and F, and E and F, respectively.

$$\angle BAP + \angle BCP = 180^\circ \qquad (1)$$
$$\angle PAD + \angle APD = 90^\circ \qquad (2)$$
$$\angle BAP + \angle PAD = 180^\circ \qquad (3)$$
$$\angle BCP + \angle EPC = 90^\circ \qquad (4)$$

Subtracting the sum of (3) and (4) from the sum of (1) and (2) gives $\angle EPC = \angle APD$. $\angle AFD = \angle APD$, and $\angle EFC = \angle EPC$. Hence $\angle AFD = \angle EFC$. Therefore DEF is a straight line.

Also solved by *Alice Walker, Blackfoot, Idaho; George Sergent, Tampico, Mexico; D. M. Shaun, Tampa, Florida; E. de la Garza, Brownsville, Texas; H. A. Obenauf, Culver, Indiana, WS Rumberg, Spokane, Washington.*

1004. *Proposed by George Sergent, Tampico, Mexico.*

To cut a sphere by a plane so that one of the segments has a volume equal to that of the cone whose vertex is the center of the sphere and whose base is the section determined by the plane in the sphere. How far is the plane from the center of the sphere?

Solved by Tillie Dantowitz, Philadelphia, Pa.

Let R be the radius of the sphere; $x =$ the distance of the center of the sphere to the cutting plane, or the altitude of the cone; $(R - x)$ = the altitude of the segment of the sphere.

The volume of the segment $= (2R+x)(R-x)^2 (\pi)/3$.

The volume of the cone $= (R^2-x^2)x(\pi)/3$.

Equating the two expressions and simplifying we get

x^2+Rx-R^2. Solving gives $x = R(5-1)/2 = .618R$.

The above expression for x shows that the cutting plane divides the radius in extreme and mean ratio.

Also solved by *Paul Mount-Campbell, Monte Vista, Col.; H. A. Obenauf, Culver, Indiana; Smith D. Turner, Cambridge, Mass.; F. A. Butter, Jr., San Jose, Calif.; E. de la Garza, Brownsville, Texas;* and the *Proposer.*

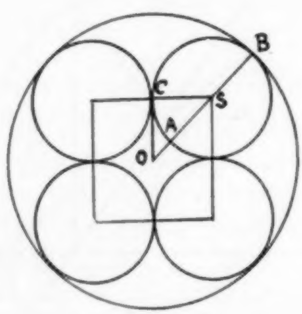
One incorrect solution was received.

1006. *Proposed by F. P. Hennessey, New York City.*

A man, in his will, left a circular plot of land of 500 acres, to be divided among his wife and four sons. Each son was to receive as much land as is contained in one of the four equal circles that can be inscribed in the circular plot; the wife to receive all that remained. What was each son's share and the wife's share?

Solved by Frances Ostewig, Lee, Ill.

Let R be the radius of the given circular plot, and r the radius of one of the inscribed circles. R is easily found to be 159.58 rd., using π = 3.1416.



Since triangle OCS is right isosceles, $OS = r\sqrt{2}$. Hence $r\sqrt{2}+r = R$. Therefore $r = R/(\sqrt{2}+1) = R(\sqrt{2}-1) = 66.103$ rd. Hence each son gets $(\pi)r^2/160$ acres = 85.782 acres, and the wife gets 156.872 acres.

Also solved by *D. M. Shaun, Tampa, Florida; Daniel Kreth, Wellman, Iowa; George Sergent, Tampico, Mexico; Tillie Dantowitz; Philadelphia, Pa. Alice Walker, Blackfoot, Idaho; R. T. McGregor, Elk Grove, Calif.; J. Murray Barbour, Aurora, N. Y.; F. A. Butter, Jr., San Jose, Calif.; Smith D. Turner, Cambridge, Mass.;* and *E. de la Garza, Brownsville, Texas.*

PROBLEMS FOR SOLUTION.

Editor: The problems selected are geometric in character, but their solutions demand a good use of algebra.

1019. *Proposed by E. de la Garza, Brownsville, Texas.*

Suggested by 1005. From any point on the circumcircle of a give triangle, lines are drawn making equal angles with the sides of the triangle. Prove the points of intersection of these lines with the sides are collinear.

1020. *Proposed by George Sergent, Tampico, Mexico.*

A cylinder bisects the volume of a sphere of radius unity, and its axis is a diameter of the sphere. Find radius of cylinder.

1021. *Proposed by Nathan Altshiller-Court, University of Oklahoma.*

The product of the distances of the vertices of a triangle from the center of an escribed circle is equal to the square of the diameter of the circle considered multiplied by the circumradius of the triangle.

1022. *Proposed by E. de la Garza, Brownsville, Texas.*

Given the sides a, b and c of a triangle, determine the amount x that must be subtracted from the length of each side, to make the triangle a right triangle. Check using $a = 40$, $b = 32$, and $c = 31$.

1023. *Proposed by George Sergent, Tampico, Mexico.*

AB is a chord of a given circle. Determine on the circle a point X, such that $(AX)^2 = AB \times XY$, XY being the perpendicular distance from X to AB.

1024. *Proposed by E. de la Garza, Brownsville, Texas.*

Suggested by 983. Given a sphere and a circular cone resting upon the same plane; the radii of the sphere and of the base of the cone being equal, the height of the cone being equal to the diameter of the sphere. Find at what distance from the summit of the cone must pass a horizontal plane to cut both solids, so that the areas of the sections are equivalent.

ANNUAL CHEMISTRY CONTEST.

At Rhode Island State College, May 12, 1928.

TIME AND PLACE: The examination will take place in Science Hall, at 8:30 a. m.

SCOPE: The ground to be covered by the examination is practically that stated in the maximum college entrance requirements for secondary schools. There will be 100 questions of a type that can be answered quickly and the time allowed to answer these will be one hour.

ELIGIBILITY: Only bona fide undergraduate students from Rhode Island high schools or preparatory schools can enter this contest. No post graduate or special student may compete or anyone who was in last year's contest.

CONTESTANTS: Each school may send a team of from three to five pupils to compete for major prizes. The sum of the three highest scores of each team will constitute the total score for that school as in the rifle shoot method, the maximum score possible being 300. Individuals from any school, however, which may not have sent a team, can compete for individual honors.

PRIZES: A new silver trophy is offered as the major prize becoming the permanent property of the high school winning it three years in succession. Individual prizes will be presented to the members of the team winning first place and to the three highest scoring students in the examination.

RESULTS of contest: The standing of the competing teams will not be published, except for those winning prizes. Individual schools, however, will be sent the record of their students and also their relative position in the list of schools.

All correspondence concerning this contest should be addressed to Prof. J. W. Ince, Chemistry Department, Rhode Island State College.

SCIENCE QUESTIONS.

CONDUCTED BY FRANKLIN T. JONES.

Questions for discussion, examination papers, disputed points may be submitted to this department. They will be published together with discussion.

Please let us know what you are working on. It will be helpful to pass the information along.

Send all communications to my home address—Franklin T. Jones, 10109 Wilbur Ave., Cleveland, Ohio.

QUESTIONS FOR SOLUTION.

514. *Proposed by William Isadore Heard, Iselin, Pa.*

In the December, 1927, issue of SCHOOL SCIENCE AND MATHEMATICS, the following question was asked in the Massachusetts Institute of Technology examination list:

"A stone is dropt over a cliff and 5.36 seconds later the sound of the stone striking the canyon floor is heard. How deep is the canyon?"

What formulas will be used in solving this problem? Is there a rule giving an approximate answer without going through a complete solution?

515. *Proposed by J. C. Packard, Brookline, Mass.*

Distinguish between energy and momentum (a) without using a formula (b) in accordance with the latest theories concerning matter and energy.

Why is momentum conserved in the collision of bodies of matter while energy is not?

[Teachers and Pupils: Please write out and send in answers to the above question by Mr. Packard. Thanks! *The Editor.*]

516. What do you think of the "New Type Question in Physics"—"In Chemistry"? Propose examples of such questions and send them in.

517. Is Frayne right?

Read the first article in SCHOOL SCIENCE AND MATHEMATICS for April, 1928 (page 345),—THE PLIGHT OF COLLEGE PHYSICS by John G. Frayne, Antioch College, and send in your comments.

SOLUTIONS AND ANSWERS.

502. *Proposed by Smith D. Turner, Cambridge, Mass.*

Per pound of water passing through a hydraulic turbine, we can get more and more energy as we increase the pressure head of the water. It is proposed to place a turbine at the bottom of a well so deep that the energy obtained from each pound of water, when converted into electricity by a generator run by the turbine, will be sufficient to electrolyze a pound of water. The resulting gases, being lighter than air, may rise through an adjoining shaft to the top, where they may be burned, the water condensed, and returned down the well. The fact that the machines used are not 100% efficient will not prevent the system from working, as the well may be made still deeper than the theoretical depth, and enough additional power developed to overcome the losses due to inefficiencies. Power may be taken from the system (a) by making the hole so deep that more electricity is generated than is needed to electrolyze the water and overcome the losses (b) from the lifting effect of the rising gases (c) from the heat generated by the burning gases at the top (d) by using the resulting superheated steam in a steam engine.

Aside from the practical difficulties involved in the above system:

1) Would the system run as described?

2) If so, from what source does it draw its energy, or would it constitute perpetual motion?

3) If it would not run, and give power as described, point out the fallacy in the above reasoning.

Answer by L. S. Guss, Austin Senior High School, Austin, Minnesota.

The system will not run as described.

It is true that the deeper the well, the more power one can get from the turbine. We are assuming, however, that the same quantity of energy is required to electrolyze water under any condition. Actually, however, this is not the case. In changing one gram of water to hydrogen and oxygen, there is an increase in volume of approximately 1800 cc. In other words, the volume of the water expands over 1800 times. Work is therefore required to push back the atmosphere by this amount. The quantity of this work will increase with increasing pressure.

Therefore, although we will obtain greater power with a greater fall of water, the increasing atmospheric pressure will cause a corresponding increase in the amount of energy required to electrolyze the water.

506. *Proposed by F. A. Vernon, Manual Training High School, Muskogee, Oklahoma.*

Will you kindly settle this question for us?

Are we not in error when we say "frost is frozen dew"?

Should we wait until the dew forms and then let it freeze will we have frost or ice?

Should you blow your breath on a mirror and freeze the moisture will ice or frost be formed?

Solution by L. S. Guss, Austin Senior High School, Austin, Minnesota.

Frost is not frozen dew. Frost is water vapor which has condensed below the freezing point. If dew forms first and then freezes, we will have a case of freezing water and hence ice will be formed.

When you blow your breath on a mirror, small droplets of water are formed thereon. If this is frozen, small pieces of ice and not frost, will be formed.

ARTICLES IN CURRENT PERIODICALS.

American Journal of Botany, March, Brooklyn Botanic Garden, Lancaster, Pa., $7.00 a year, 75 cents a copy. Studies on the Growth of Root Hairs in Solutions VI. Structural Responses to Toxic pH and Molar Concentrations of Calcium Chlorid, by Clifford H. Farr. Light Intensities Required for Growth of Coniferous Seedlings, by C. G. Bates and Jacob Roeser, Jr., Rocky Mountain Forest Experiment Station, Colorado Springs, Colo.

American Mathematical Monthly, February, Menasha, Wis., $5.00 a year, 60 cents a copy. The Analysis Situs of the Plane when the Directed Line is Taken as Element, by Jesse Douglas. Napier's Logarithms as He Developed Them, by W. D. Cairns, Oberlin College.

Education, April, The Palmer Company, Boston, $4.00 a year, 40 cents a copy. Subject Supervision, by M. W. Sloyer, Supervisor of Social Sciences, Lancaster, Pa. Is Education Responsible for Crime Wave? by A. W. Forbes, Worcester, Mass. Pre-School Education, by Wendell Sooy, Stonehurst, Pa.

Journal of Chemical Education, March, Rochester, N. Y., $2.00 a year, 35 cents a copy. A Chemistry Exhibit, by Fannie L. Bell, Ridgewood High School, Ridgewood, N. J. Chemistry in the Development of the Gas Industry, by Minor C. K. Jones, Chief Chemist,

Consolidated Gas, Electric Light & Power Co., Baltimore, Md. What to Expect of the High-School Student in Chemical Formula and Equation Writing, by O. L. Brauer, State Teachers' College, San Jose, Calif.

Journal of Geography, March, A. J. Nystrom & Co., 2240 Calumet Ave., Chicago, Ill., $2.50 a year, 35 cents a copy. Geographic Influences in the Location and Growth of the City of Memphis, by R. W. Johnson, West Tennessee State Teachers College, Memphis. Map Study for a School in Long Island, New York, by Irene J. Curnow, Clark University.

National Geographic Magazine, April, Washington, D. C., $3.50 a year, 50 cents a copy. Holidays Among the Hill Towns of Umbria and Tuscany, by Paul Wilstach. A Walking Tour Across Iceland, by Isobel Wylie Hutchison.

Popular Astronomy, April, Northfield, Minn., $4.00 a year, 45 cents a copy. The Sun Dial and its Construction, by H. B. Curtis. The Teaching of Astronomy, by Harriet W. Bigelow.

School Review, March, University of Chicago Press, Chicago, Ill., $2.50 a year, 30 cents a copy. The Junior High Schools of Kansas City, Kan., by R. L. Lyman. Making Supervision Objective, by Roswell C. Puckett, Director of High Schools, Toledo, Ohio.

Science, Grand Central Terminal, New York City, $6.00 a year, 15 cents a copy. March 30. Some Applications of Paleontology, by William Berryman Scott, Princeton University. The Contribution of Biology, Chemistry and Physics to the Newer Knowledge of Rickets, by Dr. Alfred F. Hess, New York, N. Y. Acoustics of Auditoriums, by F. R. Watson, University of Illinois.

Scientific American, April, New York, $4.00 a year, 35 cents a copy. Sharks, by David Starr Jordan, Chancellor Emeritus, Stanford University. What is Light? by Ernest O. Lawrence, Assistant Professor of Physics, Yale University and J. W. Beams, Instructor of Physics, Yale University. World Earthquake Belts, by Bailey Willis, President, Geological Society of America.

Scientific Monthly, April, The Science Press, New York, $5.00 a year, 50 cents a copy. The Chemical Elements Indispensable to Plants, by Professor Chas. B. Lipman, University of California. Botanical Explorations in the Rocky Mountains—The Lolo Trail, by Professor J. E. Kirkwood, The University of Montana. Tobacco and Scholarship, by Dr. J. Rosslyn Earp, Antioch College.

BOOKS RECEIVED.

Coal and the Coal Mines by Homer Greene. Revised Edition. Illustrations. Cloth. 225 pages. 11x17.5 cm. 1928. Houghton Mifflin Company, Boston, Mass. Price $1.24.

Plane Geometry by William W. Strader and Lawrence D. Rhoads of the Wm. L. Dickinson High School, Jersey City, N. J. Cloth. Pages xv+399. 12x18.5 cm. 1927. The John C. Winston Company, Chicago, Ill.

Teachers' Manual to Accompany New Civic Biology by George W. Hunter, Ph.D., Adjunct Professor of Biological Sciences, Pomona College, Claremont, Calif. Cloth. Pages v+105. 12x18.5 cm. 1927. American Book Company, Chicago, Ill. Price 60 cents.

Elements of Machine Design by James D. Hoffman, M.E., Head of the Department of Practical Mechanics and Director of the Practical Mechanics Laboratories, Purdue University, Lafayette, Indiana and Lynn a Scipio, M. E., Dean of Robert College School of Engineering and Professor of Mechanical Engineering, Robert College, Constantinople, Turkey. Cloth. Pages vii+327. 14.5x23 cm. 1928. Ginn and Company, Chicago, Ill. Price $3.80.

College Algebra by Kenneth P. Williams, Professor of Mathematics, Indiana University. Cloth. Pages xv+312. 13.5x20.5 cm. 1928. Ginn and Company, Chicago, Ill. Price $2.00.

An Oral Drill Book in Arithmetic by L. L. Everly, Director of Research, Public Schools, St. Paul, Minn. Cloth. Pages xi+147. 13x19 cm. 1928. The Public School Publishing Co., Bloomington, Ill.

Senior Mathematics Book I by Ernst R. Breslich, Assistant Professor of the Teaching of Mathematics, The College of Education, University of Chicago. Cloth. Pages xxii+355. 13x19.5 cm. 1928. The University of Chicago Press, Chicago, Ill. Price $1.50.

A Survey of the Achievement of Oregon Pupils in the Fundamental School Subjects by Homer P. Rainey, Ph.D. Paper. 51 pages. 17.5x25.5 cm. September, 1927, Educational Series. Published by the University, University Press, Eugene, Oregon.

The Experimental Comparison of the Relative Effectiveness of Two Sequences in Supervised Study by Harl Roy Douglass, Professor of Education and Director of University High School, University of Oregon. Paper. 45 pages. 17.5x25.5 cm. December, 1927, Educational Series. Published by the University, University Press, Eugene, Oregon.

Forests and Water in the Light of Scientific Investigation by Raphael Zon, Director, Lake States Forest Experimental Station, United States Forest Service. Paper. 106 pages. 14.5x23 cm. 1927. United States Government Printing Office, Washington. Price 20 cents.

Through Electrical Eyes by John Mills, Director of Publications, Bell Telephone Laboratories. Paper. 15.5x23 cm. 1928. Bell Telephone Laboratories, 463 West Street, New York.

BOOK REVIEWS.

The Calculus, by Robert D. Carmichael, Professor of Mathematics, University of Illinois, and James H. Weaver, Professor of Mathematics, Ohio State University. Pp. xiv+345. 21 cm. 1927. Boston. Ginn and Company. $2.80.

This book is designed for a first course in the calculus. The material supplied is sufficient for eight, or possibly ten, semester hours. This material seems to be well organized. The first chapter opens with a set of reference formulas from geometry, algebra, trigonometry, and analytic geometry. This is followed by a very interesting discussion of some of the fundamental notions of the calculus.

The aim of the authors is to pitch the discussion to the level of the student. The reviewer believes that they have accomplished this aim to a very large extent. J. M. Kinney.

Algebra, by William Raymond Longley, Professor of Mathematics, Yale University, and Harry Brooks Marsh, Head of the Department of Mathematics, Technical High School and Springfield Junior College, Springfield, Mass. Pp. ix+601. 19.5x13.5 cm. 1927. New York. Macmillan and Company.

In this revised edition the authors have brought forth a text which complies with the recommendations of the National Committee on Mathematical Requirements and with the Requirements of the College Entrance Examination Board. There are many features that are worth mentioning.

1. The notion of functionality has received some attention in connection with formulas and graphs. One whole chapter is devoted to some simple algebraic functions.

2. There are many problems of a genuine practical nature. Some of these are based on full page illustrations, such as the Map of the World, the Yale Bowl, Diagram for the Size of a Shaft, Chart for the Capacity of a Silo, Relative Distance of the Planets from the Sun, and the Seventh Street Suspension Bridge at Pittsburgh.

3. Graphs are employed throughout the texts.

4. There is much drill material and a large number of verbal problems.

5. Teachers who like a large amount of mechanical manipulation will be pleased with the abundance of material of a formal character.

6. The text includes such topics as Trigonometric Ratios, Logarithms, Arithmetic and Geometric Series, and the Binomial Theorem.

J. M. Kinney.

Modern Plane and Solid Geometry, by Webster Wells, Author of a Series of Texts on Mathematics, and Walter W. Hart, Associate Professor of Mathematics, School of Education, University of Wisconsin. Pp. ix+480. 19.5x14.5 cm. 1927. Boston. D. C. Heath and Company. $1.64. The Plane and Solid bound separately are each $1.36.

This book is designed to satisfy the needs of pupils of varying abilities. The minimum course contains all of the starred theorems of the College Entrance Examination Board together with such other theorems (also in the Boards' list) that are needed to make a logical sequence. This course is intended for pupils with but little geometrical ability.

Material for the more capable pupils is found at the end of each book. This material is gathered in groups which are labeled as Optional Topics A, B, C, and D. In addition to this material one finds about forty pages of "Additional Exercises" at the close of the Plane Geometry and eleven pages at the close of the Solid Geometry.

There is a system of cross references to this additional material making it possible for the busy teacher to select suitable problems with the assurance that preparation has already been made for their solution. There is a very large number of exercises. These consist of one, or more, exercises accompanying almost every theorem, review exercises, as well as the supplementary exercises at the end of the book.

J. M. Kinney.

Plane Trigonometry, by N. J. Lennes, Professor of Mathematics, University of Montana, and A. S. Merril, Professor of Mathematics, University of Montana, with the Editorial Cooperation of H. E. Slaught, Professor of Mathematics, University of Chicago. Pp. x+179+92. 23.5x15.5 cm. 1928. New York. Harper. With tables, $2.20; without tables, $1.60; tables alone, $1.20.

The authors state that two questions were foremost in their minds during a period of ten years in which book was in preparation, viz:

(1) Is the content and arrangement of the book naturally adapted to the type of course that is to be given?

(2) Is the book a good teaching instrument?

In order that the first question might be answered in the affirmative by a large number of teachers of trigonometry, they have supplied material which should appeal to at least three types of students.

(1) To those who wish a minimum course. They believe the first ninety pages offer enough material for this course.

(2) To those who are preparing for technical work. For this class there are sections dealing with variation of the functions, the slide rule, problems relating to physics, and finally numerous practical problems in a supplementary list.

(3) To those who are interested primarily in pure mathematics.

In order that the second question might be answered affirmatively they have devised the following features:

(1) Exercises and problems are, in most cases, provided in two or three groups labeled A, B, and C. These groups are of equal difficulty and are designed to make it possible for the teacher to give different sets in successive years or different sets to different classes.

These groups of problems are usually followed by miscellaneous

sets of greater difficulty. The problems are interesting and seem to have been carefully graded.

(2) Cumulative reviews have been provided. In these reviews the principal formulas are made to appear so that each one will be reviewed in accordance with the latest information in regard to the best spacing of such reviews.

There is a chapter devoted to the historical development of trigonometry. The tables are arranged so that they are remarkably easy to read. J. M. Kinney.

An Introduction to General Chemistry, by W. M. Blanchard, Ph.D., Professor of Chemistry, DePauw University. 1st edition. Pp. viii+588. 14.5x21x3 cm. Illustrated. Cloth. 1928. Doubleday, Doran & Co. Inc., Garden City, N. Y.

Having really read this book the reviewer is in a position to call attention to a number of outstanding features of it. In the first place it is a remarkably fine piece of teaching procedure. It is more largely inductive in method than most college texts in chemistry. Not only is there a natural and logical connection between the successive parts of the book but there is much application of the important principles after they are first taught. For example, after the modern notion of the nature of oxidation and reduction has been presented the author loses no opportunity to apply it to the recurring cases of oxidation and reduction in later parts of the book. The presentation of the molecular and atomic theories, of molecular and atomic weights and the derivation of chemical formulas is especially strong. The college teacher will find the text most teachable for the student will find it understandable and helpful.

The illustrations include a fine gallery of portraits of distinguished chemists and as many line drawings as are needed to make clear the arrangement of apparatus for the experiments that are described.

The book is thoroughly up to date in regard to the application of the new chemistry of electrons, protons, nuclei, planetry electrons to the subjects of valence, radio activity, atomic structure and isotopes. It not only presents these ideals but uses them.

The brief chapter on the hydrocarbons and their derivatives is excellently done. There are thought provoking questions at the ends of the chapters.

This text is especially recommended to those teachers who seek to train their students to learn how to think chemically rather than to amass a huge collection of chemical information. F. B. W.

Guide to the Geology of Middletown, Conn., and Vicinity, by William North Rice Ph.D., L.L.D., Professor Emeritus of Geology, Wesleyan University, and Wilbur Garland Foye, Ph.D., Professor of Geology, Wesleyan University. Pp. 127 plus list of bulletins of the State Geological and Natural History Survey of Connecticut. 15x23x1 cm. Illustrated. Paper, 1927. May be sent for 5 cents postage to teachers and scientists. $1 postpaid to others.

This is Bulletin No. 41 State of Connecticut, State Geological & Natural History Survey. To one who made many trips in his student days with Prof. Rice this bulletin is extremely interesting. The wealth of geological material in and about Middletown, due to the triple character of the rocks of the region, makes this section of the utmost interest to the geologist. Ancient crystalline rocks, whether of igneous or metamorphic origin, more recent trap rocks and sedimentary rocks may all be found within walking distance of the college. To those teachers who seek to make their field trips more interesting this bulletin is especially recommended, for it is written, trip after trip, with illustrations and with the lessons of the trips clearly taught. The wonderful pegmatite dykes with their rich stores

of rare and of gem minerals are described, the many faults, the old brownstone quarries, the changing course of the Connecticut River, the canyon of the little Westfield River with its Falls and the beauties of the Hanging Hills are among the many features of the bulletin. All teachers of geology or of earth science should procure the work if possible. F. B. W.

Oxidation-Reduction Reactions in Inorganic Chemistry, by Eric R. Jette, Ph.D., Assistant Professor of Chemistry Washington Square College, New York University. 1st edition. Pp. xvi+152. 13x19x1 cm. Cloth. 1927. List price $1.10. Century Co.

The purpose of the author of this little monograph is "to present a comprehensive discussion of oxidation-reduction reactions for the student who has had enough training in chemistry to understand ordinary chemical terminology but who has not had the benefit of a course is physical chemistry." The book gives special attention to the balancing of equations of the oxidation-reduction type and uses both the "valence change" method and the more modern "ion-electron" method. In order that the latter method may be understood the modern concepts of atomic structure and the ideas of polar and non polar compounds have been introduced. A rapid scanning of the text and a careful reading of the section on the reduction of nitric acid have given the reviewer considerable respect for the thoroughgoing character of the treatment of his subject by the author. Credit is given to Prof. V. K. Le Mer (now of Columbia University) for some of the ideas and methods of treatment. Having recently heard Prof. Le Mer lecture on Oxidation Potentials the reviewer is prepared to say that the author could hardly have had a keener contributor.

Presumably this book is for the use of second year students in inorganic chemistry or for the assistance of those in first year college chemistry as supplementary to the usual text as well as for students in qualitative analysis. College teachers of chemistry should see it.
F. B. W.

The Rise of Modern Physics by Henry Crew, Ph. D., Professor of Physics in Northwestern University. Cloth. Pages xv+356. 12.5x18.5 cm. 1928. The Williams & Wilkins Company, Baltimore. Price $5.00.

Professor Crew has here provided physics teachers with the necessary material for approaching the subject from the historical view point, which is now being advocated by many educators as a means of humanizing physics courses. In a fascinating story form he traces the gradual development of physical knowledge from the time of ancient Egypt and Babylon to the present day. After gathering together the discoveries made by the Greeks and the somewhat unimportant contributions of the Romans, the author points out the valuable work of the Arabs in numbers, astronomy, optics and mechanics, and in preserving the ancient knowledge for later generations. The history of scientific progress is followed through the middle ages by reviewing the studies of such men as Albertus Magnus, Roger Bacon, Leonardo da Vinci, Nicolaus Copernicus and John Kepler who were the fore-runners of the great trio, Galileo, Huygens, Newton. The discoveries made by these three giants in the seventeenth century mark the beginning of modern physics. Up to this time the author has followed closely the lives of the leaders but, in no sense, is the book a series of biographical sketches; it is a story of progress in scientific knowledge and understanding. However, the remainder of the book is more definitely built on several series of great related studies such as the investigation of the dispersion and refraction of light, the nature of heat, the discrete nature of matter, electromagnetism, and modern spectroscopy.

Throughout the book the author has been careful to credit each investigator with his contributions and to show his dependence upon previous discoveries as well as the effect of his work upon later investigations. He has made frequent use of both the original papers and of other histories of science. It is a book all teachers of physics will want to read. G. W. W.

An Introductory Textbook of Electrical Engineering, by John Robert Benton, Professor of Physics and Electrical Engineering in the University of Florida. Cloth. Pages xi+347. 14x23 cm. 1928. Ginn and Company, Chicago. Price $3.60.

This is a textbook prepared for use in brief elementary courses in electrical engineering. It is intended for use in beginning classes of students who are starting their technical work, and for those who want an elementary knowledge of the subject as a supplement to other engineering courses or for general education. The curriculum for students preparing to teach physics and elementary electricity should include just such a course as is covered by this text. It is devoted almost entirely to the fundamentals of the subject, the plan being to leave the special applications such as illumination, power transmission, etc., for more advanced courses for the students specializing in electrical engineering. General college physics and calculus are prerequisites for the course but the use of the calculus is limited to a very few paragraphs. In the main the mathematics consists of simple trigonometry and algebra, the entire book being about as free from mathematical language as it is possible for a book on this subject to be. The author has provided for a review of the essentials of electricity and magnetism in the first chapter, thus giving students who are somewhat rusty an opportunity for review and a few pages for handy reference.

The strongest feature of the book is its simplicity and lucidity. Nearly every page contains one or more clear diagrams or half-tones, each of which focuses attention upon a single detail of construction or theory. The language used is concise and understandable, showing that the author has unusual skill in sensing the difficulties students have in interpreting description and explanation. G. W. W.

The Earth and Its History by John Hodgdon Bradley, Jr., Ph.D., Associate Professor of Geology in the University of Montana. Cloth. Pages vii+414. 13.5x20.5 cm. 1928. Ginn and Company, 15 Ashburton Place, Boston. Price $2.60.

This book is a suitable textbook for a brief introductory course in senior high school or junior college classes in geology. It is written in simple, non-technical language and emphasizes general principles of geological processes. Judging from the many well-chosen illustrations and the artistic appearance of the book, as well as from the vivid descriptions, one of the principal aims of the author has been to make his subject interesting. The historical chapter on "The Growth of Knowledge of the Earth" is a departure from the usual content of an introductory text but this innovation will certainly meet with universal approval. G. W. W.

The Nature Almanac, A Hand Book of Nature Education edited by Arthur Newton Pack, President, The American Nature Association, and E. Laurence Palmer, Professor of Rural Education, Cornell University. Cloth. Pages viii+312. 14.5x21 cm. 1927. The American Nature Association, Washington, D. C. Price $1.00.

Teachers and students of nature study are seriously handicapped because of the unorganized condition of the subject. In many places each teacher is required to work out the course of study, organize it and find reference material. The Nature Almanac is the first of

a series designed to promote nature study and to give teachers concrete suggestions and references for their work.

Following a few pages of inspirational material which shows the need for nature study the authors have provided a nature calendar which tells the activities of plant and animal life each month. About eighty pages tell of the activities of nature associations and clubs. Another section is the report of a nature education survey which shows what is being done in each state and gives the names of teachers most active in nature work. The most valuable part of the book for most teachers is the School Nature Outline, which outlines by topics suitable material with reference for the first six grades. A valuable bibliography is appended. G. W. W.

Essentials of Junior High School Mathematics, Books One, Two and Three by Samuel Hamilton, Ph.D., LL.D., Formerly Superintendent of Schools, Allegheny County, Pa., Ralph P. Bliss, Ph.B., Chairman of Department of Mathematics, Alexander Hamilton High School, Brooklyn, N. Y., and Lillian Kupfer, Ph.D. Book One, cloth, xxiii+212 pages, 12.5x18 cm. Price 84 cents. Book Two, cloth. xxi+216 pages. 12.5x18 cm. Price 88 cents. Book Three, cloth. xxxiii+346 pages. 12.5x18 cm. Price $1.20. 1927. American Book Company, Chicago, Ill.

This is a well graded series of mathematics texts emphasizing mastery of the fundamental operations, ability to think clearly, practical applications of elementary mathematics and building a good foundation for courses in higher mathematics. Book One is mainly arithmetic but introduces the pupil to algebraic concepts through the use of the equation and the formula. Geometric concepts are introduced by construction, folding, measuring, etc. Book Two includes considerable commercial arithmetic, dealing with measurement of surfaces and volumes and extends the use of algebraic methods. A few pages are given to the Metric system. Book Three is principally algebra, but introduces demonstrative geometry and the three trigonometric ratios, sine, cosine and tangent. Graphing is taught in each book and is used frequently in creating interest and expressing results. G. W. W.

The following supplementary books and laboratory manuals are worthy of attention:

MATHEMATICS:

My Work Book in Arithmetic by Garry Cleveland Myers and Caroline Elizabeth Myers, The Harter School Supply Co., 2046 E. 71st St., Cleveland, Ohio. A very attractive work book for the second grade.

Horace Mann Supplementary Arithmetic Book II by Milo B. Hillegas, Mary G. Peabody and Ida M. Baker, J. B. Lippincott Co. A long list of good drill exercises in the fundamental processes in common and decimal fractions.

An Oral Drill Book in Arithmetic by L. L. Everly, The Public School Publishing Co., Bloomington, Ill. A new type of drill book on a much neglected phase of arithmetic work. Covers all fundamental combinations through decimal fractions. Every arithmetic teacher should see this book.

Supplementary Problems in Algebra by Herbert L. Sackett and Mary Fitzgerald, The Macmillan Co. For first year algebra, includes drill problems, new type tests and problems from Regents examination papers.

Intermediate Algebra by Elmer Schuyler, Oxford Book Co. One of the Oxford Review Series covering factoring, radicals, quadratics, progressions, logarithms, graphs and other topics.

Solid Geometry Problems and Questions by Franklin T. Jones,

The University Supply and Book Co., 10109 Wilbur Avenue, S. E., Cleveland. This is another of Mr. Jones' splendid review books; contains 845 problems and questions gleaned from a score of examining bodies.

PHYSICS:

The Harvard Tests, Elementary Physics, Form A and B by N. Henry Black and Frances M. Burlingame, Ginn and Co. Each pad contains thirty copies of the three parts of the form with directions for scoring. Each form covers the entire field of secondary school physics.

Physics Experiment Sheets by Willard B. Nelson, Globe Book Co. Sixty experiments with directions for performing, tabular forms for recording data and results and space for drawings, conclusions, etc.

CHEMISTRY:

Chemistry Experiment Sheets by Martin Mendal and Milton B. Brundage, Globe Book Co. Fifty-nine experiments with blank page opposite for student's record of observations and conclusions.

G. W. W.

WOOD CELLULOSE RIVALS COTTON.

King Cotton must abdicate in what was once considered to be the very heart of his realm, the southeastern states. From central Mississippi eastward, it costs so much to grow cotton that the high-priced product can not compete with the low-priced cellulose now being manufactured from wood and soon to be made from cornstalks. This thesis is boldly laid down by Dr. William J. Hale, director of organic chemical research of the Dow Chemical Company.

"The old practice of raising cotton in this section seems destined to obsolescence," said Dr. Hale. "The cost of growing cotton in this section is approximately ten cents per pound and yet you must face the inroads of alpha cellulose from woody fiber offered on the market at eight cents. Millions of pounds of cotton will be displaced from industrial use this year in the manufacture of rayon and nitrating paper. Even cotton linters at four cents per pound can not compete long, but possibly at two cents may still find considerable use. In other words, cotton must be driven to ten or twelve cents per pound if it is to hold its position in the textile world."

Cotton will still hold its own west of the Mississippi, where it can be produced at five or six cents per pound, Dr. Hale believes. In the Southeast, its cultivation can be continued at a profit for some years to come on the larger plantations, but the smaller farmer will do well to look at once for other crops. Dr. Hale recommended especially peanuts, which can be pressed for a high-grade food oil. Their shells also have good potentialities as industrial material. Sugar cane, where it can be grown, and sorghum to the north of the sugar cane belt, were other suggestions. Besides their yield of sugar and molasses, these plants are coming into an immense demand as sources of fiber for artificial lumber.—*Science News-Letter.*

The L. E. Knott Apparatus Company have just issued a catalogue which will be of especial service to Physics teachers. While the publishers refer to it as a reprint, it lists many new instruments with which the Physics teacher should be familiar.

It will be noted that, following the custom of this company, many of their designs are the development of ideas which practical teachers have found most useful, showing a spirit of cooperation between the teacher and the manufacturer which can but be helpful to the cause of Physics teaching.

SNOW AND STEAM FROM VOLCANO.

Mt. Lassen, America's principal active volcano, is still asleep, but the spectacular effect produced by blowing clouds of snow, mixing with the steam that the crater is continually emitting, may give the illusion of a return to activity. The effect is especially striking when it occurs near sunrise or sunset, reports R. H. Finch, associate volcanologist of the U. S. Geological Survey, whose job it is to keep his finger on the pulse of the slumbering volcano. That it is merely slumbering, and not dead, is indicated not only by the steam, but by frequent earthquakes. Sometimes several shocks occur on the same day.

Evidence of subterranean activity also comes from Glass Mountain, about 75 miles north of Lassen Peak, and in the Modoc lava beds. Fairly recent lava flows are to be found nearly all the way between Glass Mountain and Lassen Peak, and Forest Service officials in the vicinity report about half an acre of land covered with pumice which is very hot. By digging but a little way into the pumice much higher temperatures are reached, and near the pumice bed is a deep fissure emitting steam.

With George L. Collins, of the National Park Service, Mr. Finch attempted to make temperature measurements and to take photographs on the mountain, about 7,850 feet high, during January, but were driven back by a heavy snow storm. They are now planning to conduct further explorations in the spring.

According to the Indians the heat of Glass Mountain has been known for many years, and earthquakes originating in the mountain and accompanied by rattling noises have been noted by Forest Service officials for at least 15 years.—*Science News-Letter.*